1 ARITHMETIC

THIS CHAPTER INCLUDES

1. Ratios, Variations and Proportions
2. Time Value of Money and Annuity – Simple and Compound Interest
3. Arithmetic Progression and Geometric Progression
4. Time and Distance

OBJECTIVE QUESTIONS

QUESTIONS AND ANSWERS OF JUNE 2013

1. If x : y = 3 : 4, y : z = 5 : 6 and z : w = 7 : 8, then x: y : z : w is
   (a) 3:5:7:8 (b) 3:4:6:8
   (c) 105: 140: 168: 192 (d) 3 : 4 : 24 : 192 **(1 mark)**
   Answer: (c)
2. The average monthly consumption of petrol for a car for 12 months is 160 litre, if the average monthly consumption for first 8 months is 145 litre, then the average monthly consumption of petrol for the last 4 months is
   (a) 190 litre (b) 165 litre
   (c) 180 litre (d) 175 litre **(1 mark)**
   Answer: (a)
3. If 9 men working 10 hours daily can complete a job in 10 days, then 15 men working 6 hours daily shall complete same job in
   (a) 6 days (b) 8 days
   (c) 10 days (d) none of the above **(1 mark)**
   Answer: (c)

4. In 25 years at 8% p.a. simple interest, a sum becomes ₹ 4,629. The amount of sum is
   (a) ₹ 1,534 (b) ₹ 1,453
   (c) ₹ 1,435 (d) ₹ 1,543 **(1 mark)**
   Answer: (d)
5. A quantity p varies directly as t and another quantity q varies inversely as t. When t = 2, p + q = 1 and when t = 3, p + q = 8.When t = 4, p + q =
   (a) 8.5 (b) 9
   (c) 9.5 (d) 8 **(1 mark)**
   Answer: (c)

QUESTIONS AND ANSWERS OF SEPTEMBER 2014

1. The monthly salaries of two persons are in the ratio of 3:5. If each receives an increase of ₹ 20 in salary, the ratio is altered to 13:21. Find the respective salaries
   (a) ₹ 300, ₹ 500 (b) ₹ 130, ₹ 210
   (c) ₹ 240, ₹ 400 (d) ₹ 400, ₹ 240 **(1 mark)**
   Answer: (c)
2. If the two numbers 20 and x+2 are in the ratio of 2:3; Find x
   (a) 14 (b) 28
   (c) 32 (d) 29 **(1 mark)**
   Answer: (b)
3. The simple interest on a sum of money of the end of 8 years is $\frac{2}{5}$th of the sum itself. Find the rate percent p.a.
   (a) 15% (b) 5%
   (c) 20% (d) 10% **(1 mark)**
   Answer: (b)
4. In what time will be the S.I. on ₹ 900 at 6% be equal to S.I. on ₹ 540 for 8 years at 5%.
   (a) 4 years (b) 10 years
   (c) 8 years (d) 6 years **(1 mark)**
   Answer: (a)

5. The fourth term and seventh term of G.P. are 24 and 192 respectively. Find the sum of its first 10 terms.
   (a) 2,192 (b) 3,069
   (c) 9,063 (d) 192 **(1 mark)**
   Answer: (b)
6. If the sum of first n terms of the series 5, 9, 13, 17,---------- is 275, then find n.
   (a) n = 17 (b) n = 9
   (c) n = 11 (d) n = 5 **(1 mark)**
   Answer: (c)

QUESTIONS AND ANSWERS OF DECEMBER 2014

1. What must be subtracted from each of the numbers 17, 25, 31, 47 so that the remainders may be in proportion.
   (a) 12 (b) 6
   (c) 9 (d) 3 **(1 mark)**
   Answer: (d)
2. The ratios of 200 gm to 2 kg. is:
   (a) 100:1 (b) 10:1
   (c) 1:1 (d) 1:10 **(1 mark)**
   Answer: (d)
3. If the 5^{th} and 11^{th} terms of an A.P. be 41 and 20, then find its first term and the sum of first 11 terms.
   (a) 65, 410.5 (b) 55, 412.5
   (c) 50, 412.5 (d) 53, 400.5 **(1 mark)**
   Answer: (b)
4. If a,b,c be the p^{th}, q^{th} and r^{th} term of an A. P. then a(q - r)+ b(r - p)+ c (p - q) equals:
   (a) a
   (b) 0
   (c) b
   (d) c **(1 mark)**
   Answer: (b)

QUESTIONS AND ANSWERS OF MARCH 2015

1. The Ratio of 5kg 55gm to 35kg 50gm:
   (a) 5:7 (b) 1,011:7,010
   (c) 111:710 (d) None of these **(1 mark)**
   Answer: (b)
2. If A:B = 3:4, B:C = 2:5 then A:B:C:
   (a) 3:4:5 (b) 3:4:10
   (c) 4:3:10 (d) 3:4:8 **(1 mark)**
   Answer: (b)
3. Gulshan Kumar borrows ₹ 300 at 5% and ₹ 450 at 6% at the same time and on the condition that the whole loan will be repaid when the total interest amounts to ₹ 126. The loan will have to be repaid after how many years:
   (a) 2 (b) 3
   (c) 4 (d) 5 **(1 mark)**
   Answer: (b)
4. To find out the total compound interest accrued on a sum of money after 5 years, which of the following information given in the statements P and Q will be sufficient?
   P: The sum was ₹ 20,000
   Q: The total amount of simple interest on the sum after 5 years was ₹ 4,000.
   (a) Only P is sufficient (b) Only Q is sufficient
   (c) Either P or Q is sufficient (d) Both P & Q are needed.
   (1 mark)
   Answer: (c)
5. If $\frac{\sqrt{a+\sqrt{b}}}{\sqrt{a-\sqrt{b}}} = \frac{2}{1}$ then $\frac{a+b}{a-b}$ is equal to:
   (a) 5/4 (b) 4/5
   (c) 3 (d) None of these. **(1 mark)**
   Answer: (a)

QUESTIONS AND ANSWERS OF JUNE 2015

1. If A : B = 2 : 3, B : C = 4 : 5, then A : C =
   (a) 6 : 7 (b) 7 : 6
   (c) 8 : 15 (d) 15 : 8 **(1 mark)**
   Answer: (c)
2. The ratio of two numbers is 11 : 15. The sum of 3 times the first number and twice the second number is 630. The H.C.F. of the number is:
   (a) 10 (b) 12
   (c) 15 (d) None of these. **(1 mark)**
   Answer: (a)
3. If $a^{1/3} + b^{1/3} + c^{1/3} = 0$, then $(a + b + c)^3$:
   (a) 3 abc (b) 27 abc
   (c) -27 abc (d) None of these. **(1 mark)**
   Answer: (b)
4. The simple interest on ₹ 10 for 4 months at the rate of 3 paise per rupee per month is:
   (a) ₹ 1.20 (b) ₹ 12
   (c) ₹ 120 (d) ₹ 1,200 **(1 mark)**
   Answer: (a)

QUESTIONS AND ANSWERS OF SEPTEMBER 2015

1. A sum of money doubles itself in 10 years. The number of years it would triples itself is
   (a) 25 years (b) 15 years
   (c) 20 years (d) 10 years **(1 mark)**
   Answer: (c)
2. The fourth proportional of ₹ 5, ₹ 3.50, 150 gm is:
   (a) 100 gm (b) 105 gm
   (c) 125 gm (d) None of these. **(1 mark)**
   Answer: (b)

3. If 2A = 3B = 4C, then A : B : C is:
   (a) 2 : 3 : 4 (b) 4 : 3 : 2
   (c) 6 : 4 : 3 (d) 3 : 4 : 6. **(1 mark)**
   Answer: (c)
4. A person borrowed ₹ 500 at the rate of 5% per annum S.I. what amount will he pay to clear the debt after 4 years?
   (a) ₹ 200 (b) ₹ 550
   (c) ₹ 600 (d) ₹ 700 **(1 mark)**
   Answer: (c)
5. A sum of money at compound interest amounts to thrice itself in 3 years. In how many years will it be 9 times itself in?
   (a) 12 (b) 9
   (c) 6 (d) 8 **(1 mark)**
   Answer: (c)
6. If x varies inversely with y and if y = 3, then x = 8. The value of y when x = 2 are:
   (a) 24 (b) 18
   (c) 12 (d) None of these. **(1 mark)**
   Answer: (c)

QUESTIONS AND ANSWERS OF DECEMBER 2015

1. The C.I. on ₹ 40,000 at 10% p.a. for 1 year when the interest is payable quarterly is:
   (a) ₹ 4,000 (b) ₹ 4,100
   (c) ₹ 4,152.51 (d) None of these **(1 mark)**
   Answer: (c)
2. If A:B = 2:3 B:C = 4:5 then A:C
   (a) 6 : 7
   (c) 7 : 6
   (c) 8 : 15
   (d) 15 : 8 **(1 mark)**
   Answer: (c)

3. The third proportional of 1 hour 20 minutes 1 hour 40 minutes is:
   (a) 1 hours 50 minutes (b) 2 hours
   (c) 2 hours 5 minutes (d) 2 hours 25 minutes **(1 mark)**
   Answer: (c)
4. If 15% of x is the same as 20% of y, then x : y is
   (a) 3 : 4 (b) 4 : 3
   (c) 17 : 16 (d) 16 :17 **(1 mark)**
   Answer: (b)
5. A fraction which bears the same ratio to $\frac{1}{27}$ that $\frac{3}{11}$ does to $\frac{5}{9}$ is:
   (a) $\frac{1}{55}$ (b) 55
   (c) $\frac{1}{11}$ (c) 14 **(1 mark)**
   Answer: (a)
6. The simple interest on ₹ 10 for 4 months at the rate of 3 paise per rupee per months is:
   (a) ₹ 1.20 (b) ₹ 12
   (c) ₹ 120 (d) ₹ 1,200 **(1 mark)**
   Answer: (a)
7. To find out the total compound interest accrued on a sum of money after 5 years, which of the following information given in the statements P and Q will be sufficient?
   P: The sum was ₹ 20,000.
   Q: The total amount of simple interest on the sum after 5 year was ₹ 4,000.
   (a) Only P is sufficient (b) Only Q is sufficient
   (c) Either P or Q is sufficient (d) Both P & Q is necessary.
   (1 mark)
   Answer: (d)
8. The fourth term and seventh term of G.P. are 24 and 192 respectively. Find the sum of its first 1o terms.
   (a) 2,192 (b) 3,069
   (c) 9,063 (d) 192 **(1 mark)**
   Answer: (b)

9. If $x \propto a^2$, then $a \propto$ …….
   (a) x^4 (b) $\sqrt{x}$
   (c) $\frac{1}{\sqrt{x}}$ (d) None of these **(1 mark)**
   Answer: (b)
10. If $x^2 \propto yz$, $y^2 \propto zx$, $z^2 \propto xy$, then the product of three constant of variation is:
   (a) 0 (b) 1
   (c) 3 (d) xyz **(1 mark)**
   Answer: (b)

QUESTIONS AND ANSWERS OF MARCH 2016

1. If A:B = 2:3, B:C = 4:5, then A:C =
   (a) 6:7 (b) 7:6
   (c) 8:15 (d) 15:8 **(1 mark)**
   Answer: (c)
2. The inverse ratio of $1\frac{3}{5} : 2\frac{1}{4}$ is_____.
   (a) 32:45 (b) 45:32
   (c) 18:5 (d) 5:18 **(1 mark)**
   Answer: (c)
3. The ratio $\frac{5}{3} : 2\frac{1}{4}$ is:
   (a) Ratio of lesser inequality (b) Ratio of greater inequality
   (c) 20:9 (d) 5:27 **(1 mark)**
   Answer: (a)
4. The ratio of 5kg. 55gm. to 35kg. 50gm.:
   (a) 5:7
   (b) 1,011:7010
   (c) 111:710
   (d) None of these **(1 mark)**
   Answer: (b)

5. The ratio is 1 year 6 months : 2 years : 2 years 6 months:
   (a) 3:4:5 (b) 2:3:5
   (c) 2:4:5 (d) None of these **(1 mark)**
   Answer: (a)
6. $\left[\frac{1}{2}+\frac{1}{3}\right] : \left[\frac{1}{2}\times\frac{1}{3}\right]$
   (a) 2:3 (b) 3:2
   (c) 5:1 (d) 1:5 **(1 mark)**
   Answer: (c)
7. The mean proportional of 4X and $16X^3$ is _____.
   (a) 10×2 (b) 12×2
   (c) 8×2 (d) 64×4 **(1 mark)**
   Answer: (c)
8. If $\frac{1}{5} : \frac{1}{x} = \frac{1}{x} : \frac{1}{1.25}$ the value of x is:
   (a) 1.5 (b) 2
   (c) 2.5 (d) 3.5 **(1 mark)**
   Answer: (c)
9. If $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$, then $\frac{a+b+c}{c}$ is equal to:
   (a) 7 (b) 2
   (c) $\frac{1}{2}$ (d) $\frac{1}{7}$ **(1 mark)**
   Answer: (b)
10. The total number of factors of 210 (excluding 1 and 210) is _____.
    (a) 14 (b) 16
    (c) 18 (d) 20 **(1 mark)**
    Answer: (a)
11. If $\frac{\sqrt{a+\sqrt{b}}}{\sqrt{a-\sqrt{b}}} = \frac{2}{1}$ then $\frac{a+b}{a-b}$ is equal to:
    (a) 5/4 (b) 4/5
    (c) 3 (d) None of these **(1 mark)**
    Answer: (a)

QUESTIONS AND ANSWERS OF JUNE 2016

1. The fourth proportional of 0.2, 0.12 and 0.3 is:
   (a) 0.13 (b) 0.15
   (c) 0.18 (d) 0.8 **(1 mark)**
   Answer: (c)
2. The third proportional to $(x^2 - y^2)$ and $(x - y)$ is:
   (a) $\frac{x + y}{x - y}$ (b) $\frac{x - y}{x + y}$
   (c) $x + y$ (d) $(x - y)$ **(1 mark)**
   Answer: (b)
3. A fraction which bears the same ratio to $\frac{1}{27}$ that $\frac{3}{11}$ does to $\frac{5}{9}$ is:
   (a) $\frac{1}{55}$ (b) 55
   (c) $\frac{1}{11}$ (d) $\frac{3}{11}$ **(1 mark)**
   Answer: (a)
4. The mean proportional between $\sqrt{11} - \sqrt{5}$ and $13\sqrt{11} + 19\sqrt{5}$ is:
   (a) $\sqrt{33} - \sqrt{15}$ (b) $\sqrt{33} + \sqrt{15}$
   (c) $\sqrt{11} + \sqrt{5}$ (d) None of these **(1 mark)**
   Answer: (b)
5. A fraction which bears the same ratio to $\frac{1}{27}$ and $\frac{3}{11}$ does to $\frac{5}{9}$ is:
   (a) $\frac{1}{55}$
   (b) 55
   (c) $\frac{1}{11}$
   (d) $\frac{3}{11}$ **(1 mark)**
   Answer: (a)

QUESTIONS AND ANSWERS OF DECEMBER 2016

3. Choose the correct answer:

(a) If $\frac{x}{2} = \frac{y}{3}$, then x : y is equal to

(i) 3 : 2
(ii) 2 : 3
(iii) 5 : 6
(iv) 6 : 5 **(1 mark)**

Answer: (ii) 2 : 3

(b) If $x \propto y$ and when x = 5, y = 5 then the value of x when y = 10 is

(i) 100
(ii) 0
(iii) 1
(iv) 10 **(1 mark)**

Answer: (iv) 10

QUESTIONS AND ANSWERS OF JUNE 2017

1. Choose the correct answer:

(i) Three numbers are in the ratio 5: 7: 12 and the sum of the first and third numbers is greater than the second by 50. The sum of the three numbers is

(a) 130 (b) 120
(c) 128 (d) 125 **(2 marks)**

Answer: (b) 120

(ii) In a certain time ₹ 1,400 becomes ₹ 1,848 at 8% p.a. simple interest. When ₹ 2,100 will become ₹ 2,604 at the same time, the rate of interest is

(a) 8.2%
(b) 7%
(c) 10%
(d) 6% **(2 marks)**

Answer: (d) 6%

(iii) The year by which a sum of rupees would be 1.21 times of itself at 10% per annum C.I. is

(a) 2 years (b) 3 years
(c) 3.5 years (d) 2.5 years **(2 marks)**

Answer: (a) 2 years

(vi) Which term 128 is, of the progression 1, 2, 4, 8, ...?

(a) 8^{th} term (b) 7^{th} term
(c) 9^{th} term (d) 10^{th} term **(2 marks)**

Answer: (a) 8^{th} term

(viii) $y = A + B$ where A and B vary directly and inversely respectively with x; $x = 1$ when $y = 11$ and $x = 2$ when $y = 13$. The value of y when $x = 3$ is

(a) 15 (b) 17
(c) 19 (d) 20 **(2 marks)**

Answer: (b) 17

2. State whether the following statement is True or False:

(i) $1 + 3 + 5 + 7 + ... + (2n - 1) = n^2$. **(1 mark)**

Answer: (i) True

QUESTIONS AND ANSWERS OF DECEMBER 2017

1. Choose the correct answer:

(i) The mean proportional between 9 and 25 is

(a) 17
(b) 15
(c) 225
(d) 16 **(2 marks)**

Answer: (b) 15

(ii) p varies inversely as q. If $p = 2$ then $q = 3$. Find p if $q = 2$.

(a) 3
(b) 4
(c) 1
(d) 0 **(2 marks)**

Answer: (a) 3

(iii) A person deposits ₹ 2,000 at 6% p.a. simple interest for 3 years. The amount he will get back after 3 years is
(a) ₹ 2,300
(b) ₹ 2,400
(c) ₹ 2,360
(d) ₹ 2,350 **(2 marks)**
Answer: (c) ₹ 2,360

(iv) Find the 10^{th} term of the A. P. 3, 6, 9, 12, 15, ...
(a) 20
(b) 25
(c) 23
(d) 30 **(2 marks)**
Answer: (d) 30

(v) The product of 3 terms in a G.P. is 125. The middle term is
(a) 3
(b) 4
(c) 5
(d) 6 **(2 marks)**
Answer: (c) 5

2. State whether the following statements are True or False:
(iii) The series 1, 11, 111, 1111,..... is an AP series.
(iv) The 7^{th} term of the progression 3, – 9, 27, ... is 2187.
(vi) The true discount on a bill of ₹ 1,040 due for 6 months at 8% p.a. is ₹ 40. **(1 × 3 = 3 marks)**

Answer:
(iii) False
(iv) True
(vi) True.

QUESTIONS AND ANSWERS OF JUNE 2018

1. Choose the correct answer:
(i) The ratio of work done by (x + 2) men in (x – 2) days to that of (x–1)men in (x + 1) days is 4 : 5, the value of x is
(a) ± 4
(b) 6

(c) 4
(d) 8 **(2 marks)**

Answer: (c) 4

(ii) The mean proportional between 2 and 8 is
(a) 4
(b) 16
(c) 3
(d) 1 **(2 marks)**

Answer: (a) 4

(iii) The simple interest (SI) on ₹ 100 at the rate of 5% p.a. for 5 years is
(a) ₹ 10
(b) ₹ 5
(c) ₹ 1
(d) ₹ 25 **(2 marks)**

Answer: (d) ₹ 25

(iv) The 7^{th} term of the Arithmetic Progression (AP) 7, 10, 13, 16,.....is
(a) 28
(b) 22
(c) 25
(d) 20 **(2 marks)**

Answer: (c) 25

2. State whether the following statements are *True* or *False*:

(iv) The geometric mean of 3 and $\frac{1}{3}$ is – 1. **(1 mark)**

Answer:

(iv) False

QUESTIONS AND ANSWERS OF DECEMBER 2018

1. Choose the correct answer:

(i) If $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$, then the value of $\frac{a+b+c}{c}$ is
(a) 4
(b) 2

(c) 7
(d) 14 **(2 marks)**

Answer: (b) 2

(ii) If p varies directly as q and if q = 2 then p = 4. If p = 2, the value of q is
(a) 1.5
(b) 2
(c) 1
(d) 3 **(2 marks)**

Answer: (c) 1

(iii) A man deposited a sum of money to a bank at 9% simple interest p.a. The total interest that he will get at the end of 5 years is ₹ 1,620. The deposited amount is
(a) ₹ 6,000
(b) ₹ 4,000
(c) ₹ 3,600
(d) ₹ 4,400 **(2 marks)**

Answer: (c) ₹ 3,600

(iv) The mean proportional between 4 and 16 is
(a) 8
(b) 10
(c) 9
(d) ± 8 **(2 marks)**

Answer: (d) ± 8

(v) The 6^{th} term of an A.P. 2, 5, 8 ..... is
(a) 18
(b) 16
(c) 17
(d) 19 **(2 marks)**

Answer: (c)17

(vi) The 7^{th} term of the series 16, 8, 4, 2, ... is
(a) $\frac{1}{8}$
(b) $\frac{1}{4}$

(c) $\frac{1}{2}$

(d) $\frac{1}{16}$ **(2 marks)**

Answer: (b) $\frac{1}{4}$

2. State whether the following statements are *True* or *False*:
   (i) Some money is distributed between Amal and Ashoke in the ratio 3 : 5. If Amal receives ₹ 72, then Ashoke receives ₹ 108. **(1 mark)**
   (iii) Speed (s) is inversely proportional to time (*t*). Then st = constant. **(1 mark)**

Answer:
(i) False
(iii) True

QUESTIONS AND ANSWERS OF JUNE 2019

1. Choose the correct answer:
   (i) Six years before the ratio of the ages of two sisters Mitali and Sonali is 2 : 3. If the present age of Mitali is 30 years, then the present age of Sonali is
   (a) 28 years
   (b) 42 years
   (c) 36 years
   (d) 48 years **(2 marks)**
   Answer: (b) 42 years
   (ii) If *I* is the simple interest, *R* is the rate % p.a., *T* is the time period, *P* being the principal, then *I* is expressed as
   (a) $I = 100\ (PRT)$
   (b) $I = \frac{100}{PRT}$

(c) $I = \frac{PRT}{100}$

(d) $I = PRT$ **(2 marks)**

Answer: (c) $I = \frac{PRT}{100}$

(iii) Find the compound interest on ₹ 4,000 for 2 years at 5% p.a.

(a) ₹ 400

(b) ₹ 405

(c) ₹ 415

(d) ₹ 410 **(2 marks)**

Answer: (d) ₹ 410

(v) If p varies inversely as q, it is written as

(a) $p \alpha \frac{1}{q}$

(b) $q \alpha \frac{1}{p}$

(c) $p^2 \alpha q^2$

(d) $\frac{1}{p^2} \alpha \frac{1}{q^2}$ **(2 marks)**

Answer: (a) $p \alpha \frac{1}{q}$

2. State whether the following statements are *True* or *False*:

(iv) The sum of first 6 terms of the arithmetic progression (A.P.) 6, 4, 2, 0, ... is 12. **(1 mark)**

Answer: False

(vi) The geometric mean of two quantities p and q is pq. **(1 mark)**

Answer: (vi) False

QUESTIONS AND ANSWERS OF DECEMBER 2019

1. (a) Choose the correct answer:

(i) Two numbers are in the ratio 5:7. If the sum of the numbers is 192, then the greater number is

(a) 112

(b) 102

(c) 116
(d) 108 **(2 marks)**
Answer: (a) 112

(ii) x varies inversely as y and if x = 3 then y = 4. If y = 6 then the value of x is
(a) 3
(b) 1
(c) 2
(d) 2.5 **(2 marks)**
Answer: (c) 2

(iii) A sum will be double itself at a simple interest p.a. in 8 years. The simple interest is
(a) 10%
(b) 10.5%
(c) 12%
(d) 12.5% **(2 marks)**
Answer: (d) 12.5%

(iv) If (x – 1), (x + 1) and (2x + 3) are in A.P., then the value of x is
(a) – 2
(b) 0
(c) 2
(d) 4 **(2 marks)**
Answer: (b) 0

(v) The 8^{th} term of the series 256, 128, 64, ... is
(a) 2
(b) 4
(c) 8
(d) 16 **(2 marks)**
Answer: (a) 2

2. State whether the following statements are *True* or *False:*
(i) The mean proportional between 2 and 8 is 4. **(2 marks)**
(ii) If 1+2+3+.....+n = 231 then the value of n is 21. **(2 marks)**

Answer:
(i) True
(ii) True

QUESTIONS AND ANSWERS OF DECEMBER 2022

1. If x varies Inversely as y and if x = 3 then 4 If y = 6, then the value of x is
   (a) 1
   (b) 2
   (c) 3
   (d) 2.5 (1 mark)
   Answer: (b) 2
2. Salaries of 5 members of a joint family follow arithmetic progression, the average salary being ₹ 30000. If the lowest salary is ₹ 10000, then highest salary in ₹ will be.
   (a) 45,000
   (b) 60,000
   (c) 50,000
   (d) 70,000 **(1 mark)**
   Answer: (c) 50,000
3. Find the sum of 6 terms of the G.P 3, 9, 27..
   (a) 1092
   (b) 1029
   (c) 1290
   (d) 1209 **(1 mark)**
   Answer: (a) 1092
4. In a definite time period ₹ 1200 becomes ₹ 1560 at 10% per annum simple interest. Find the principal amount that will become 2232 at 8% per annum in the same time period:
   (a) 1850
   (b) 1800
   (c) 1875
   (d) 2050 **(1 mark)**
   Answer: (b) 1800
5. The 25^{th} term of the series 1,3,5,7,.....is
   (a) 48
   (b) 59

(c) 49
(d) 47 **(1 mark)**

Answer: (c) 49

6. In some years, ₹ 1500 becomes ₹ 1980 at 8% simple interest per annum. Find the number of years.
(a) 6
(b) 3.5
(c) 4
(d) 5 **(1 mark)**

Answer: (c) 4

7. The average ages of male and female employees of a company are 40 and 35 years respectively. The average age of all employees is 38 years. Find the percentage of female employees.
(a) 40%
(b) 60%
(c) 35%
(d) 50% **(1 mark)**

Answer: (a) 40%

8. The compound interest on ₹ 2000 at 10% per annum for 2 years and 6 month compounded annually will be :
(a) ₹ 450
(b) ₹ 600
(c) ₹ 538
(d) ₹ 541 **(1 mark)**

Answer: (c) ₹ 538

9. If (x + 1), (x + 5) and (x + 19) are in continued proportion, then the value of x is :
(a) 14
(b) 0.6
(c) 0.26
(d) 4 **(1 mark)**

Answer: (b) 0.6

10. The ratio of 500 liters and 1000 grams is:
    (a) cannot be determined
    (b) ½
    (c) 3
    (d) 2 **(1 mark)**

    Answer: (a) cannot be determined

QUESTIONS AND ANSWERS OF JUNE 2023

1. How many terms of the series 1 + 3 + 9 + 27 + ... amount to 3280?
    (a) 10
    (b) 12
    (c) 8
    (d) 6 **(2 marks)**

    Answer:
    (c) 8
2. A car takes 5 hours to travel 100 km. How much time would be required to cover the distance 90 km. with the same speed?
    (a) 4.3 hr.
    (b) 4.5 hr.
    (c) 4.2 hr.
    (d) 4.8 hr. **(2 marks)**

    Answer:
    (b) 4.5 hr.
3. The income-tax varies as the square of the excess of the income of a man over ₹ 5,00,000 and on an income of ₹ 6,00,000 it amounts to ₹ 5,000. Find how much a man has to pay under this system, whose income is ₹ 8,00,000.
    (a) ₹ 40,000
    (b) ₹ 45,000
    (c) ₹ 42,000
    (d) ₹ 54,000 **(2 marks)**

    Answer:
    (b) ₹ 45,000

4. What sum of money will yield ₹ 1,407 as interest in $1\frac{1}{2}$ years at 14% simple interest per annum?
   (a) ₹ 6,700
   (b) ₹ 7,600
   (c) ₹ 6,600
   (d) ₹ 6,500 **(2 marks)**
   Answer:
   (a) ₹ 6,700
5. If a, b, c are in A.P., then $\frac{a-b}{b-c}$ is equal to:
   (a) 1
   (b) –1
   (c) ±1
   (d) $\frac{b}{a}$ **(2 marks)**
   Answer:
   (a) 1
6. The ratio of salaries of two persons in a factory are in the ratio 5 : 7. If the 2nd person received ₹ 12,000 more than the 1st, then the salary of the 2nd person is:
   (a) ₹ 42,000
   (b) ₹ 36,000
   (c) ₹ 48,000
   (d) ₹ 60,000 **(2 marks)**
   Answer:
   (a) ₹ 42,000
7. A rope of length 480 cm. is divided into four parts whose lengths are proportional to 2, 5, 6, 7. The length of the maximum part is:
   (a) 144 cm.
   (b) 152 cm.
   (c) 180 cm.
   (d) 168 cm. **(2 marks)**
   Answer:
   (d) 168 cm.

8. A sum of money becomes ₹ 3,364 at 16% compounded annually for 2 years. The sum of money is:
   (a) ₹ 2,200
   (b) ₹ 1,800
   (c) ₹ 3,800
   (d) ₹ 2,500 **(2 marks)**

   Answer:
   (d) ₹ 2,500

QUESTIONS AND ANSWERS OF DECMBER 2023

1. The age of two persons are in the ratio 4:3. If the age of the younger person is 8 years less than the other, then the age of the elder person is:
   (a) 40 (b) 32
   (c) 44 (d) 36 **(2 marks)**

   Answer:
   (b) 32
2. If 15 men working 8 hours a day can complete a piece of work in 25 days, find how many days will be taken by 12 men working 10 hours a day to complete it.
   (a) 20 (b) 30
   (c) 25 (d) 15 **(2 marks)**

   Answer:
   (c) 25
3. Mean proportional between 0.20 and 0.05 is:
   (a) 0.1 (b) 0.2
   (c) 0.3 (d) 0.12 **(2 marks)**

   Answer:
   (a) 0.1
4. A person deposited ₹ 1,000 in a bank at 4% p.a. simple interest. What will be the amount after 10 years?
   (a) ₹ 1,500
   (b) ₹ 1,400

(c) ₹ 1,300
(d) ₹ 1,600 **(2 marks)**

Answer:
(b) ₹ 1,400

5. A sum of ₹ 10,000 deposited C.I. becomes double after 5 years. After 20 years it will become:
(a) ₹ 1,20,000
(b) ₹ 2,00,000
(c) ₹ 1,50,000
(d) ₹ 1,60,000 **(2 marks)**

Answer:
(d) ₹ 1,60,000

6. The 7^{th} term of an A.P. 2, 5, 8, 11, ... is:
(a) 18
(b) 14
(c) 20
(d) 15 **(2 marks)**

Answer:
(c) 20

7. Find 8^{th} term of the series, 128, 64, 32, ...
(a) $\frac{1}{4}$ (b) -1
(c) $-\frac{1}{2}$ (d) 1 **(2 marks)**

Answer:
(d) 1

8. A takes 4 hours to cover a distance 80 km and B takes 5 hours to cover the same distance. How much distance would A cover more than B in 15 hours?
(a) 40 km (b) 60 km
(c) 55 km (d) 45 km **(2 marks)**

Answer:
(b) 60 km

QUESTIONS OF JUNE 2024

1. In 30 litres of adulterated milk, the ratio of the volumes of milk and water is 7 : 3. What volume of water should be added to it to make the ratio of milk and water 3 : 7?
   (a) 20 (b) 40
   (c) 25 (d) 30 **(2 marks)**
2. The mean proportional between 2 and 8 is:
   (a) 4 (b) 5
   (c) 3 (d) 6 **(2 marks)**
3. p varies directly with the cube root of q and p is 4 when the value of q is 8. Find the value of (q + 1) when the value of p is 6.
   (a) 25 (b) 28
   (c) 32 (d) 36 **(2 marks)**
4. If the interest rate is 6% p.a., for what sum of money (in ₹) will the difference between compound interest and simple interest for 2 years is ₹ 13.5?
   (a) 5,730
   (b) 5,370
   (c) 3,750
   (d) 3,570 **(2 marks)**
5. A bank offers 4% nominal interest with quarterly compounding. What is the effective rate of interest?
   (a) 4.01%
   (b) 4.02%
   (c) 4.06%
   (d) 4.08% **(2 marks)**
6. The sum of first 50 natural numbers is:
   (a) 1275
   (b) 2550
   (c) 2549
   (d) 2500 **(2 marks)**

7. A car travels some distance at a speed 8km/hour and return at a speed 12 km/hour. If the total time taken is 15 hours, what is the distance (in km)?
   (a) 48
   (b) 60
   (c) 56
   (d) 72 **(2 marks)**
8. If product of the first three terms of a G.P. is 64, the middle term is:
   (a) 2 (b) 8
   (c) 6 (d) 4 **(2 marks)**

PRACTICAL QUESTIONS

2009 - June [1] Answer the following:
Choose the correct option showing the proper reasons/calculations.

(a) Let marks obtained by Ram, Rahim and Jadu be A, B, and C respectively. Given A : B = 1:2, B:C = 3:4. The combined ratio A : B : C is
   (i) 1 : 2 : 4,
   (ii) 3 : 6 : 8,
   (iii) 1 : 6 : 8,
   (iv) None of them. **(3 marks)**

(b) If $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{2}{1}$ then $\frac{a+b}{a-b}$ is equal to
   (i) 5/4
   (ii) 4/5
   (iii) 3
   (iv) None of them **(3 marks)**

Answer :

(a) If A : B = 1 : 2 and B : C = 3 : 4
Then A:B:C = 1 × 3 : 2 × 3 : 2 × 4
= 3 : 6 : 8

(b) $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{2}{1}$

Using componendo and Dividendo.

$\frac{\sqrt{a}+\sqrt{b}+(\sqrt{a}-\sqrt{b})}{\sqrt{a}+\sqrt{b}-(\sqrt{a}-\sqrt{b})} = \frac{2+1}{2-1}$

$\frac{2\sqrt{a}}{2\sqrt{b}} = \frac{3}{1}$

Squaring both sides, we get

$\frac{a}{b} = \frac{9}{1}$

using again componends and Dividendo

$\frac{a+b}{a-b} = \frac{10}{8} = \frac{5}{4}$

2009 - June [2] Answer the following :

(a) If $\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{a+b}$ then show that $(b-c)(x-a) + (c-a)(y-b) + (a-b)(z-c) = 0$. **(4 marks)**

(b) A person borrowed ₹ 10,000 at some simple interest rate for 2 years. After expiry of one year he borrowed another ₹ 20,000 at 1% lower interest rate for 1 year. At the end he paid fully ₹ 33,000. Find the rate of interest at which he borrowed first. **(4 marks)**

Answer :

(a) $\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{a+b} = k$ (let)

$\therefore x = k(b+c)$

$y = k(c+a)$

$z = k(a+b)$

Now,

L.H.S.

$= (b-c)(x-a) + (c-a)(y-b) + (a-b)(z-c)$

$= (b-c)\{(k(b+c)-a\} + (c-a)\{k(c+a)-b\} + (a-b)k(a+b)-c\}$

$= k(b^2-c^2) - (ab-ac) + k(c^2-a^2) - (bc-ab) + k(a^2-b^2) - (ac-bc)$

$= k(b^2-c^2+c^2-a^2+a^2-b^2) - (ab-ac+bc-ab+ac-bc)$

$= 0$

$=$ R.H.S.

(b) Let r be the required rate of interest

P = ₹ 10,000

Simple Intt. for 2 years on ₹ 10,000 at r% intt.

$= \frac{10,000 \times r \times 2}{100} = 200r$

Since after one year he borrowed another 20,000 hence principle is ₹ 30,000 and at (r – 1)% intt. rate

$= \frac{20,000 \times (r - 1) \times 1}{100} = 200\,(r - 1)$

Total interest received in 2 years = 33,000 – 30,000 = ₹ 3,000

$\therefore$ 200r + 200(r – 1) = 3,000

200r + 200r – 200 = 3,000

400r = 3200 ; $r = \frac{3,200}{400} = 8\%$

2009 - June [4] Answer the following :

(b) The total expenses of a boarding house varies partly with the number of boarders and partly fixed. The total expenses are ₹ 10,000 for 25 boarders and ₹ 11,500 for 30 boarders. Find the fixed expenses.

(3 marks)

Answer :

Let the expenses be x

& variable expenses by y

$\therefore$ x + 25y = 10,000 ..... (1)

x + 30y = 11,500 ..... (2)

– – –

5y = 1,500

y = 300

Putting the value of y in eq. (i)

x + 25 × 300 = 10,000

or, x = 10,000 – 7,500

$\therefore$ x = 2,500

$\therefore$ Fixed expenses = 2,500

Variable exp. per boarder = 300

2009 - Dec [1] Answer the following:
Choose the correct option showing the proper reasons/calculations.

(a) The number to be added to each term of the ratio 3 : 7 to make it 1: 2 is
  (i) 2,
  (ii) 1,
  (iii) 3,
  (iv) None of these. **(3 marks)**

(b) The average of 7 numbers is 27. If one number is included, the average becomes 25. The included number is
  (i) 11,
  (ii) 10,
  (iii) 12,
  (iv) None of these. **(3 marks)**

(c) The time in which a sum of money becomes double at 10% p.a., simple interest is
  (i) 8 years,
  (ii) 10 years,
  (iii) 12 years,
  (iv) None of these. **(3 marks)**

Answer :

(a) Let the number added is x.

Now $\frac{3 + x}{7 + x} = \frac{1}{2}$

or $6 + 2x = 7 + x$

or $x = 1$ **Option (ii)**

(b) Let the number included is x

Given averages of seven number = 27

$\frac{\sum x}{7} = 27$

or $\sum x = 7 \times 27$

$= 189$

i.e. Total of Seven numbers = 189

Again Given $\frac{189 + x}{8} = 25$

or 189 + x =200

or x = 11 **Option (i)**

(c) Let the sum of money = P
and hence Amount = 2P

Now $A = P\left(1 + \frac{rt}{100}\right)$

or, $2p = P\left(1 + \frac{10.t}{100}\right)$

or, $2 = 1 + \frac{10t}{100}$

or, $2 = 1 + \frac{t}{10}$

or, t = 10 years **Option (ii)**

2009 - Dec [2] Answer the following:

(b) In a liquid mixture 20% is water and in another mixture water is 25%. These two mixtures are mixed in the ratio 5 : 3. Find the percentage of water in the final mixture. **(4 marks)**

Answer :

Let in first liquid ratio of water to others = 20% : 80%
= 1 : 4

Proportion of water = $\frac{x}{5}$

Similarly, in second liquid, ratio of water and others = 25% : 75% = 1:3

$\therefore$ Proportion of water = $\frac{y}{4}$

Now given $\frac{\frac{x}{5}}{\frac{y}{4}} = \frac{5}{3}$

$$\frac{4x}{5y} = \frac{5}{3}$$

$$\frac{x}{y} = \frac{25}{12}$$

$\therefore$ x : y = 25 : 12

2009 - Dec [3] Answer the following:

Choose the correct option showing necessary reasons/ calculations.

(c) Given a varies *as bx + c.* Value of *a* is 3 when *b* = 1, *c* = 2 and is 5 when *b* = 2, *c* = 3. The value of *x* would be

(i) –1, (ii) 2. (iii) 3. (iv) none of these. **(3 marks)**

Answer :

Given a varies as bx + c

i.e. a = k (bx + c) where k = a Constant

when b = 1 and c = 2 then a = 3

3 = k (x + 2) ---------- (i)

When b = 2 and c = 3 then a = 5

5 = k (2x + 3) ------ (ii)

Divide on (1) by (ii) we get

$$\frac{K(x+2)}{K(2x+3)} \quad \frac{3}{5}$$

5x + 10 = 6x + 9

x = 1 **option (iv)**

2010 - June [1] Answer the following:

Choose the correct option showing the proper reasons/calculations.

(a) If x is the mean proportional between x – 2 and x + 6 then the value of x is

(i) 4

(ii) 3

(iii) 2

(iv) None of these **(3 marks)**

(b) Of the five numbers the average of first four numbers is 8 and the average of the last four numbers is 6. Then the difference of the first and the fifth number is

(i) 6
(ii) 8
(iii) 10
(iv) None of these **(3 marks)**

Answer :

(a) Given $\frac{x-2}{x} = \frac{x}{x+6}$

$x^2 = (x-2)(x+6)$
$x^2 = x^2 + 6x - 2x - 12$
$4x - 12 = 0$
$4x = 12$
$x = 3$ **option (ii)**

(b) Let the five numbers are x_1, x_2, x_3, x_4, & x_5

Given $\frac{x_1 + x_2 + x_3 + x_4}{4} = 8 \Rightarrow x_1 + x_2 + x_3 + x_4 = 32$

$\Rightarrow x_2 + x_3 + x_4 = 32 - x_1$ ---- (i)

and $\frac{x_2 + x_3 + x_4 + x_5}{4} = 6 \Rightarrow x_2 + x_3 + x_4 + x_5 = 24$

$\Rightarrow x_2 + x_3 + x_4 = 24 - x_5$ ---- (ii)

Solving (i) & (ii)
$32 - x_1 = 24 - x_5$
$x_1 - x_5 = 32 - 24$
$= 8$ **option (ii)**

2010 - June [2] Answer the following:

(a) Divide ₹ 6,200 in 3 parts such that the interest for the three parts for 2, 3 and 5 years respectively at 5% simple interest p.a. are same. **(4 marks)**

(b) A dealer mixes two varieties of teas costing ₹ 100 per kg. and ₹160 per kg. in the proportion 5:1. He sold the 6 kg. mixture at the rate of ₹ 120 per kg. Find his profit. **(4 marks)**

Answer :

(a) 1st part is x, 2nd part is y, 3rd part is z
$\therefore x + y + z = 6,200$

Interest on 1st part = $x \times \frac{5}{100} \times 2 = \frac{x}{10}$

Interest on 2nd part = $y \times \frac{5}{100} \times 3 = \frac{3y}{20}$

Interest on 3rd part = $z \times \frac{5}{100} \times 5 = \frac{z}{4}$

$\therefore \frac{x}{10} = \frac{3y}{20} = \frac{z}{4} = K$

$\therefore x = 10K, y = \frac{20K}{3}, z = 4K$

$\therefore 10K + \frac{20K}{3} + 4K = 6{,}200 \Rightarrow K = 300$

1st part = $10 \times 300 = 3{,}000$, 2nd part = $20 \times \frac{300}{3} = 2{,}000$, 3rd part = 4,300

= 1,200

(b) Let us calculate the cost of mixture by Urery formula of weighted avg.

$\therefore$ Cost of mixture = $\frac{5 \times 100 + 1 \times 160}{5 + 1} = ₹\ 110$

$\therefore$ Cost of 6 kg mixture = ₹ 6×110 = ₹ 660

and selling price of 6 kg mixture = ₹ 6×120 = ₹ 720

$\therefore$ Profit = 720 − 660

= ₹ 60

2010 - June [3] Answer the following:

(e) The area of a circle varies directly with square of its diameter. Area of the circle is 38.5 sq. cm when diameter is 7 cm. If diameter of the circle is 1 cm then area of the circle in sq.cm is

(i) 5.5/7 (ii) 11/7 (iii) 22/7 (iv) None of these **(3 marks)**

Answer :

Let the area of circle = A

diameter of the circle = d

Given $A \propto d^2$

or $A = k \cdot d^2$ [where k is a constant]

Given A = 38.5 sq.cm and d = 7 cm

$\therefore A = k \cdot d^2$

$38.5 = k \cdot 7^2$

or $k = \frac{5.5}{7}$

Again when d = 1cm and k = $\frac{5.5}{7}$ cm

Then A = $\frac{5.5}{7} \times 1^2 = \frac{5.5}{7}$ sq. cm **option (i)**

2010 - June [4] Answer the following:

(c) The volume of a gas varies directly as the absolute temperature and inversely as pressure. When the pressure is 15 units and the temperature is 260 units the volume is 200 units. What will be the volume when the pressure is 18 units and the temperature is 195 units? **(3 marks)**

Answer :

Let volume = V

Pressure = P

and Absolute Temperature = T

Given $V \propto T$

and $V \propto \frac{1}{P}$

or $V = K \frac{T}{P}$ where K = Constant

When P = 15, T = 260 then V = 200

$200 = K \frac{260}{15} \Rightarrow K = \frac{150}{13}$

Hence, when P = 15 and T = 260

Then, $V = \frac{150}{13} \times \frac{195}{18} = 125$ units

2010 - Dec [1] Answer the following :

Choose the correct option showing the proper reasons/ calculations :

(a) If $P = \frac{4}{5}Q$ and $Q = 2\frac{1}{2}R$, then $P : R$ is

(i) 1 : 2 (ii) 2 : 1

(iii) 4 : 5 (iv) none of these **(3 marks)**

(b) A person drove his car 40 km at an average speed of 20 km per hour and next 60 km at an average speed of 30 km per hour. Then his average speed in his whole journey of 100 km is
(i) 25 km/h
(ii) 20 km/h
(iii) 30 km/h
(iv) none of these **(3 marks)**

(c) Time in which ₹ 5,000 will be the amount ₹ 6,000 at simple interest @ 5% p.a. is
(i) 2 years (ii) 5 years
(iii) 4 years (iv) none of these **(3 marks)**

Answer :

(a) $\frac{P}{R} = \frac{P}{Q} \times \frac{Q}{R} = \frac{4}{5} \times \frac{5}{2} = \frac{2}{1} =$ P : R = 2 : 1 Ans, (ii)

(b) Whole distance covered = 40 + 60 = 100 km

Time taken = $\frac{40}{20} + \frac{60}{30}$ = 4 hrs.

So average speed in whole distance = $\frac{\text{distance}}{\text{time}} = \frac{100}{4}$

= 25 km/hr Ans. (i)

(c) Given Time = t years , rate (r) = 5%

Principal (p) = ₹ 5,000

Amount (A) = ₹ 6,000

Hence Simple Interest = A – P

= ₹ 6,000 – ₹ 5,000 = ₹ 1,000

Now time = $\frac{S.I \times 100}{p \times r} = \frac{1,000 \times 100}{5,000 \times 5}$ = 4 years **option (iii)**

2010 - Dec [2] Answer the following :

(a) A dealer mixed two varieties of tea having cost ₹ 1,200 and ₹ 2,500 per kg each in such a way that he can gain 20% by selling the resultant mixture at ₹ 1,800 per kg. Find the proportion in which the two types of teas are mixed. **(4 marks)**

Answer :

Let the proportion of two types of teas are x_1 & x_2 and cost price of mixture = $\frac{100}{120}$ x 1800 = ₹ 1,500 hence we can rorite

$$1{,}500 = \frac{1200x_1 + 2500x_2}{x_1 + x_2}$$

$1500(x_1 + x_2) = 1200x_1 + 2500x_2$

$1500\ x_1 + 1500x_2 = 1200\ x_1 + 2500x_2$

$300\ x_1 = 1000x_2$

$$\frac{X_1}{X_2} = \frac{10}{3}$$

Hence required proportion is 10:3

2010 - Dec [4] Answer the following :

(a) The expense of a boarding house are partly fixed and partly varies with the number of boarders. The charge is ₹ 70 per head when there are 20 boarders and ₹ 60 per head when there are 40 boarders. Find the charge per head when there are 50 boarders. **(3 marks)**

Answer :

Let Y = a + bx ---------- (i)

Where x = No. of boarders

and y = charges per head.

If x = 20, Y = 70

from (1) 70 = a + 20b ---------- (ii)

again x = 40 y = 60

from (1) 60 = a + 40b ---------- (iii)

Solving (ii) & (iii) we get

a + 20b = 70

a + 40b = 60

- 20b = 10

b = -1/2

∴ a = 80

$\therefore$ Hence from (i) $y = 80 - {}^{1}/_{2x}$

If $x = 50$

They $Y = 80 - ½ \times 50 = 55$

2011 - June [1] Answer the following :

Chose the correct option showing the proper reasons/calculations.

(a) If $2 - x$, $3 - x$, $5 - x$ and $7 - x$ are in proportion, then the value of x is

(i) 1,

(ii) -1,

(iii) 2,

(iv) None of these. **(3 marks)**

(b) The average of 10 numbers is 21. If an additional number is included the average becomes 20. The additional number is

(i) 10,

(ii) 5,

(iii) 3,

(iv) None of these. **(3 marks)**

Answer :

(a) $\therefore$ The given expression are in proportion.

$\therefore$ The product of extreme terms = product of middle terms

$\therefore (2 - x)(7 - x) = (3 - x)(5 - x)$

$\therefore 14 - 2x - 7x + x^2 = 15 - 3x - 5x + x^2$

$\therefore 14 - 15 = -8x + 9x$

$-1 = x$

$\therefore x = -1$ **Option (ii)**

(b) The average of 10 numbers = 21

$\therefore$ Total $= 10 \times 21 = 210$

Let the Additional number be x

$\therefore$ Total + x = Average x Total no

$\therefore 210 + x = 20 \times 11$

$210 + x = 220$

$\Rightarrow x = 10$ **Option (i)**

2011 - June [2] Answer the following :

(a) Due to fall in rate of interest from 12% to 10% per annum in 4 years, home loan amount of a person decreases by ₹ 4,800. Find the home loan he took first. **(4 marks)**

(b) At what ratio sugar at ₹ 30 per kg be mixed with sugar at ₹ 35 per kg to produce a mixture making profit 25% when sold at ₹ 40 per kg ? **(4 marks)**

Answer :

(a) P = Home loan the person took first

n yrs = time period, i_1 = first rate of interest, i_2 = second rate of interest

Then $4800 = (P + Pn\, i_1) - (P + Pn\, i_2) = Pn(i_1 - i_2) = P.4\ (12/100 - 10/100)$

i.e. P = ₹ 60,000.

(b) X = 30 ₹/kg

Y = 35 ₹/kg

Profit % = 25%

Selling price = 40 ₹/kg

Let Cost Price be = x

$x + \frac{25}{100} x = 50$

$x = \frac{40 \times 100}{125} = 32$ ₹/kg.

$x \times 30 + y \times 35 = (x + y) \times 32$

$30x + 35y = 32x + 32y$

$3y = 2x$

$\frac{3}{2} = \frac{x}{y}$

3 : 2

2011 - June [3] Answer the following :

(c) If c varies directly as x + b, c = 8 when b = 2 and c = 10 when b = 3 then value of x is

(i) 0,

(ii) 1,

(iii) 2,

(iv) None of these. **(3 marks)**

Answer :

$c \propto (x + b)$ i.e, $c = k (x + b)$, $k \neq 0$

Thus $8 = k (x + 2)$ and $10 = k (x + 3)$

i.e $\frac{8}{10} = \frac{x + 2}{x + 3} \Rightarrow x =$ **2 option (iii)**

2011 - Dec [1] Answer the following:

Choose the correct option showing the proper reasons/calculations.

(a) Two numbers are in the ratio of 3 : 4. If 10 is subtracted from both of them then the ratio becomes 1 : 3. The numbers are

(i) 9 and 12

(ii) 12 and 16

(iii) 15 and 20

(iv) None of these **(3 marks)**

(b) A person drove his car 50 km at an average speed of 20 km/h. He drove first 30 km of his journey at an average speed of 60 km/h. Then average speed of last 20 km is

(i) 40 km/h

(ii) 25 km/h

(iii) 10 km/h

(iv) None of these **(3 marks)**

(c) For a sum of money to become $2\frac{1}{4}$ times of itself in 5 years, the rate of interest is

(i) 25%

(ii) 30%

(iii) 35%

(iv) None of these **(3 marks)**

Answer :

(a) Let the number be x

$\therefore$ According to the question.

$$\frac{3x - 10}{4x - 10} = \frac{1}{3}$$

$\Rightarrow$ $9x - 30 = 4x - 10$

$\Rightarrow$ $9x - 4x = -10 + 30$

$5x = 20$

$x = 4$

$\therefore$ the ratio will be, $3x : 4x = 3 \times 4 : 4 \times 4 = 12 : 16$ **Option (ii)**

(b) Total distance = 50 km

Average speed = 20 km/hr

$\therefore$ Total time taken = $\frac{\text{Total distance}}{\text{Average Speed}}$

$= \frac{50 \text{ km} \times \text{hr}}{20 \text{ km}} = \frac{5 \text{ hr}}{2} = 2.5 \text{ hr}$

1st Journey covered distance = 30 km

Average speed = 60 km/hr

Time taken = $\frac{30 \text{ km} \times \text{hr}}{60 \text{ km}}$

$= \frac{1}{2} \text{ hr}$

$\therefore$ Now Rest time = $\frac{5}{2}$ hr - $\frac{1}{2}$ hr = 2 hr

Now again Rest distance = 20 km

Time = 2 hr

$\therefore$ Average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{20 \text{ km}}{2 \text{ hr}} = 10$ km/hr **Option (iii)**

(c) Let the Principal be x

$\therefore$ Amount will be = $\frac{9}{4}$ x

time = 5 years

Since we know that

$P = \frac{A}{1 + rt}$

$x = \frac{\frac{9}{4}x}{1 + 5r}$

$\Rightarrow \quad 1 + 5r = \frac{9}{4}$

$5r = \frac{9}{4} - 1$

$5r = \frac{9 - 4}{4}$

$5r = \frac{5}{4}$

$r = \frac{1}{4}$

$\therefore$ rate $= \frac{1}{4} \times 100 = 25\%$ **Option (i)**

2011 - Dec [2] Answer the following:

(a) If $\frac{\alpha}{q - r} = \frac{\beta}{r - p} = \frac{\gamma}{p - q}$ then prove that $\alpha + \beta + \gamma = 0 = p\alpha + q\beta + r\gamma$.

(4 marks)

Answer:

$\frac{\alpha}{q - r} = \frac{\beta}{r - p} = \frac{Y}{p - q} = k$ (let)

$\alpha = k (q - r)$ .....(i) $\beta = k (r - p)$.....(ii) $Y = k (p - q)$ .....(iii)

Adding (i), (ii) & (iii)

$\alpha + \beta + Y = k (q - r) + k (r - p) + k (p - q)$

$= kq - kr + kr - pk + pk - kq$

$\alpha + \beta + y = 0$

Hence proved

Again

$p \alpha + q\beta + rq = pk (q - r) + qk (r - p) + rk (q - q)$

$= kpq - pkr + kqr - kqp + prk - rqk$

$p\alpha + q\beta + rq = 0$

Hence proved.

2011 - Dec [3] Answer the following:

Choose the correct option showing proper reasons/calculations.

(c) Let $A - k$ varies directly as B where k is constant. If $A = 750$ then $B = 500$. If $A = 1,175$ then $B = 1,350$. If $A = 550$ then B will be

(i) 100
(ii) 200
(iii) 250
(iv) None of these **(3 marks)**

Answer :

$A - k \propto B$

$A - k = Bt$ (t is a constant)

$750 - k = 500t$

$(-)\ 1175 - k = 1350t$

(+) (–)

$-425 = -850t$

$t = ½$

If A = 550, then, 550 – 500 = B/2

B = 100 **option (i)**

2012 - June [1] Answer the following :

Choose the correct option showing the proper reasons/calculations.

(a) 10 years before, the ages of father and son were in the ratio 5:2. If at present their total age is 90 years, the present age of the son is
(i) 40 years
(ii) 25 years
(iii) 30 years
(iv) None of these **(3 marks)**

(b) If the speed of a car to go uphill is 20 km/hr and down is 30 km/hr, then average speed of the car is (in km/hr)
(i) 23
(ii) 24
(iii) 25
(iv) None of these **(3 marks)**

Answer :

(a) Let the father's present age = x
And son's present age = y

Given, $\frac{x-10}{y-10} = \frac{5}{2}$

Or, 2x - 20 = 5y - 50

Or, 2x - 5y = - 30 .....(i)

And x + y = 90 (given) .....(ii)

Solving equation (i) and (ii), we get,

x = 60

y = 30

Therefore, the present age of son is 30 yrs. **Option (iii)**

(b) Average speed is calculated with the help of Harmonic mean (H.M.).

Therefore, Average Speed $= \frac{2}{\frac{1}{20} + \frac{1}{30}}$ = 24 km/hr **Option (ii)**

2012 - June [2] Answer the following:

(a) Two vessels contain mixtures of milk and water in the ratio 5:1 and 9:1. They are mixed together in the ratio 1:5. Find the ratio of milk and water in the final mixture. **(4 marks)**

(b) An amount of money at certain rate of simple interest per annum becomes ₹ 2,400 in 4 years and ₹ 2,500 in 5 years. Find the rate of interest p.a. **(4 marks)**

Answer:

(a) Let 1 litre of mixture of first vessel be mixed with 5 Litres of mixture of second vessel.

1 Litre of first vessel contains $1 \times \frac{5}{6} = \frac{5}{6}$ litre of milk and $1 \times \frac{1}{6} = \frac{1}{6}$ litre of water.

5 Litre of second vessel contains $5 \times \frac{9}{10} = \frac{9}{2}$ litre of milk and $5 \times \frac{1}{10} = \frac{1}{2}$ litre of water.

So, in the final mixture, milk : water $= \left(\frac{5}{6} + \frac{9}{2}\right) : \left(\frac{1}{6} + \frac{1}{2}\right) = 8 : 1$

(b) Let P be the initial sum invested

Given A = 2,400 in 4 years

Therefore, $A = P\left(1 + \frac{rt}{100}\right)$

$2{,}400 = P\left(1 + \frac{4r}{100}\right)$ .....(i)

Again, Given, A = 2,500 in 5 years

Therefore, $2{,}500 = P\left(1 + \frac{5r}{100}\right)$ .....(ii)

Dividing (i) by (ii)

$\frac{2{,}400}{2{,}500} = \frac{P(1 + 0.04r)}{P(1 + 0.05r)}$

$\frac{24}{25} = \frac{1 + 0.04r}{1 + 0.05r}$

$24 + 1.20\,r = 25 + r$

$0.2\,r = 1$

$r = \frac{1}{0.2} = 5\%$ p.a.

2012 - June [3] Answer the following :

Choose the correct option showing proper reasons/calculations.

(b) If $(a + b) \propto (a - b)$ and when a = 6, b = 2, then for b = 3, the value of a is

(i) 6

(ii) 9

(iii) 12

(iv) None of these **(3 marks)**

Answer:

Since $(a + b)\ \alpha\ (a - b)$

Therefore, $a + b = k(a - b)$

$\frac{a + b}{a - b} = \frac{k}{1}$

by applying, compenendo & dividendo rule,

$$\frac{a + b + a - b}{a + b - a + b} = \frac{k + 1}{k - 1}$$

$$\frac{2a}{2b} = \frac{k+1}{k-1}$$

$$\frac{a}{b} = \frac{k+1}{k-1}$$

for a = 6, b = 2

$$\frac{6}{2} = \frac{k + 1}{k - 1}$$

Therefore, k = 2

Therefore, for b = 3, and k = 2

$$\frac{a}{3} = \frac{2 + 1}{2 - 1}$$

a = 9 **Option (ii)**

2012 - June [4] Answer the following :

(c) The total expenses of a boarding house are partly fixed and the rest varies as the number of boarders. The charges is ₹ 100 per head when there are 25 boarders and ₹ 80 when there are 50 boarders. Find the number of boarders for which the total expense will be ₹ 7000.

(3 marks)

Answer:

Fixed Cost = C_1, Total Cost = C, No. of boarders = n

Therefore, $C = C_1 + kn$

Hence, $2,500 = C_1 + 25\ k$ and $4,000 = C_1 + 50k$

$C_1 = 1,000$, k = 60

Therefore $C = 1,000 + 60n$

Therefore, When C = 7,000, n = 100.

The no. of boarders is 100.

Alternative solution

Variable cost per unit =

$$\frac{\text{Change in total cost}}{\text{Change in units}} = \frac{(50 \times 80) - (25 \times 100)}{(50 - 25)} = \frac{(4,000 - 2,500)}{25}$$

= 60 per boarder

Fixed Cost per unit = Total Cost – Variable Cost
= 4,000 – (50 × 60) = 1,000
If Total Cost = 7,000 therefore total Variable Cost = 6,000
No. of boarders = $\frac{6,000}{60}$ = 100 boarders

2012 - Dec [1] Answer the following:
Choose the correct option showing the proper reasons/calculations.

(a) The average of 4 numbers is 13. If one number is excluded the average becomes 15. The excluded number is
(i) 5
(ii) 6
(iii) 7
(iv) none of these. **(3 marks)**

(b) The compound ratio of x : 2, 2 : 3 and 3 : y is
(i) y : x
(ii) x : y
(iii) 2 : 5
(iv) None of these. **(3 marks)**

(c) A sum of ₹ 1,200 amounts to ₹ 1,536 in 2 years at simple interest. The rate of interest per annum is
(i) 14%
(ii) 12%
(iii) 11%
(iv) None of these. **(3 marks)**

Answer :

(a) (iii) Excluded number = (4 × 13) - (3 × 15) = 7

(b) (ii) The compound ratio x × 2 × 3 : 2 × 3 x y = x : y

(c) (i) $\frac{1200 \times R \times 2}{100}$ = 1536 – 1200

24R = 336

R = 14 ∴ R = Rate of Interest = 14 %

2012 - Dec [2] Answer the following:

(b) A number is added to each of the numbers 7,15,21 and 37 so that the resulting numbers are in proportion. Find the number added.

(4 marks)

Answer :

Let the number to added be x.

$$\frac{7 + x}{15 + x} = \frac{21 + x}{37 + x}$$

or, $(21 + x)(15 + x) = (37 + x)(7 + x)$

or, $315 + 21x + 15x + x^2 = 259 + 37x + 7x + x^2$

or, $36x + x^2 - 44x - x^2 = 259 - 315$

or, $-8x = -56$

or, $x = 7$

$\therefore$ 7 will be added each of the numbers.

2012 - Dec [3] Answer the following:

Choose the correct option showing proper reasons/calculations.

(e) Given A = B + C, when B αx^2 and C αx^3. If A = 0 when x = 1 and A = 2 when x = −1, then A in terms of x is

(i) $2x^2 - x^3$

(ii) $x^2 - 2x^3$

(iii) $x^2 - x^3$

(iv) None of these. **(3 marks)**

Answer:

$A = B + C = k_1 x^2 + k_2 x^3$ [assuming $B = k_1 x^2$ & $C = k_2 x^3$]

$A = 0, x = 1 \Rightarrow k_1 + k_2 = 0$ and $A = 2, x = 1 \Rightarrow k_1 - k_2 = 2$

Hence, solving the above two $k_1 = 1$ and $k_2 = -1$,

So $A = B + C = A x^2 - x^3$.

2016 - Dec [1] Answer the questions:

(a) A sum of money invested at compound interest amounts to ₹ 10,816 at the end of second year and to ₹ 11,248.64 at the end of third year, find the rate of interest.

(c) Two numbers are in the ratio of 2 : 7 and if 9 be added to each of them, the sums become in the ratio 1 : 2. Find the numbers.

(5 × 2 = 10 marks)

Answer:

(a) $A = P\left(1 + \frac{C}{100}\right)^n$

$10,816 = P\left(1 + \frac{C}{100}\right)^2$ _____(1)

$11,248.64 = P\left(1 + \frac{C}{100}\right)^3$ _____(2)

Dividing (2) by (1) we have,

$\frac{11,248.64}{10,816} = \left(1 + \frac{C}{100}\right)$

$\frac{C}{100} = \frac{11,248.64}{10,816} - 1$

$\frac{C}{100} = \frac{432.64}{10,816}$

C = 4%.

(c) Let the two numbers be 2x and 7x

Then, 2x + 9 : 7x + 9 = 1: 2

or $\frac{2x + 9}{7x + 9} = \frac{1}{2}$

Cross multiplying, we get ,

4x +18 = 7x + 9

7x – 4x = 18 – 9

or 3x = 9

$\therefore$ x = 3

Hence, the two numbers are (2 × 3) and (7 × 3)

i.e. 6 and 21

2016 - Dec [2] Answer the questions:

(a) A man deposits ₹ 10,000 at the end of each year in a bank which pays 5% p.a. compound interest. If the installments are allowed to accumulate, what will be the total accumulation at the end of 10 years? (Given $(1.05)^{10} = 1.62889$ (approximately)).

(c) Find the ratio of ${}^5P_2 : {}^5C_2$. **(3 × 2 = 6 marks)**

Answer:

(a) Future Value $= 10{,}000 \left[\frac{(1 + 0.05)^{10} - 1}{0.05}\right]$

$= 10{,}000 \left[\frac{(1.05)^{10} - 1}{0.05}\right]$

$= 10{,}000 \left[\frac{1.62889 - 1}{0.05}\right]$

$= \frac{10{,}000 \times 0.62889}{0.05}$

$= ₹\ 1{,}25{,}778$

(c) ${}^5P_2 : {}^5C_2 = \frac{{}^5P_2}{{}^5C_2}$

$= \frac{\frac{n!}{(n - r)!}}{\frac{n!}{(n - r)!r!}} = r! = 2! = 2$

2016 - Dec [7] (d) A sum of ₹ 10,000 is invested for simple interest at the rate of 10% per annum for 3 years. Find the amount received as interest after 3 years. **(1 mark)**

Answer:

Interest $= 10{,}000 \times \frac{10}{100} \times 3 = ₹\ 3{,}000$

2017 - June [3] Answer the questions:

(b) If – 15, – 25 are respectively the 11^{th} and 16^{th} terms of an A.P., then find the sum of first 20 terms of the A.P. **(4 marks)**

(d) Find the sum of n terms of the series 2 + 22 + 222 + ... **(4 marks)**

Answer:

(b) $T_n = a + (n - 1)d$

$T_{11} = a + (11 - 1)d = a + 10d$

$\therefore$ $a + 10d = -15$ _____ (1)

$T_{16} = a + 15d$

$\therefore$ $a + 15d = -25$ _____ (2)

Subtracting (2) from (1) we have,

$$\begin{array}{r} a + 10d = -15 \\ -a - 15d = +25 \\ \hline -5d = 10 \end{array}$$

or $d = -2$

Also $a + 10d = -15$

$\therefore \quad a - 20 = -15$

or $a = 5$

Now $S_{20} = \frac{20}{2}[2 \times 5 + (20 - 1) \times (-2)]$

$S_{20} = 10\,[10 - 38] = -280$

(d) Let S denote the required sum.

i.e. $S = 2 + 22 + 222 + \ldots\ldots\ldots\ldots$ to n terms

$= 2\,(1 + 11 + 111 + \ldots\ldots\ldots$ to n terms)

$= \frac{2}{9}\,(9 + 99 + 999 + \ldots\ldots$ to n terms)

$= \frac{2}{9}\,\{(10 - 1) + (10^2 - 1) + (10^3 - 1) + \ldots\ldots + (10^n - 1)\}$

$= \frac{2}{9}\,\{(10 + 10^2 + 10^3 + \ldots\ldots + 10^n) - n\}$

$= \frac{2}{9}\,\{10\,(1 + 10 + 10^2 + \ldots\ldots + 10^{n-1}) - n\}$

$= \frac{2}{9}\,\{[10\,(10^n - 1)/(10 - 1)\} - n\}$

$= \frac{2}{81}\,(10^{n+1} - 10 - 9n)$

$= \frac{2}{81}\,(10^{n+1} - 9n - 10)$

2017 - Dec [3] Answer the following questions:

(a) Monthly income ratio of two persons is 5:6 and their monthly expenditure ratio is 3:4. If each saves ₹ 4,000 per month, find their monthly incomes.

(b) A person invests ₹ 1,00,000 on compound interest for 2 years at 10% p.a. Calculate the amount that he will get back.

(c) The sum of n terms of an A.P. is $3n^2 + 5n$. Find the number of the term which is equal to 152. **(4 × 3 = 12 marks)**

Answer:

(a) Let monthly income two be 5x and 6y and monthly expenditure be 3y and 4y

Savings will be $5x - 3y = 4,000$ ----------(1)

and $6x - 4y = 4,000$ ----------(1)

Multiply Equation (1) by 4 and Equation (2) by 3

$$\begin{array}{rl} & 20x - 12y = 16,000 \\ (-) & 18x - 12y = 12,000 \\ & (+) \quad (-) \\ \hline & 2x = 4,000 \\ \hline \end{array}$$

$x = 2,000$

Incomes $= 5 \times 2,000$ and $6 \times 2,000$

$= ₹ 10,000$ and $₹ 12,000$

(b) Amount on Maturity $= 1,00,000\ (1 + 0.1)^2$

$= ₹ 1,21,000.$

(c) $S = 3n^2 + 5$

$S_1 = 3 + 5 = 8 = a_1$

$S_2 = 3 \times 2^2 + 5 \times 2 = 22 = a_1 + a_2$

$a_2 = 22 - a_1 = 22 - 8 = 14$

$d = a_2 - a_1 = 14 - 8 = 6$

Let 152 be month term, then,

$152 = a + (m - 1)\ d$

$152 = 8 + (m - 1)\ 6$

$(m - 1)\ 6 = 152 - 8$

$m - 1 = \frac{144}{6}$

$m = 24 + 1 = 25$

$\therefore$ 152 is 25^{th} term.

2018 - June [3] Answer the following questions:

(a) If $x \propto y$, then prove that $x^2 + y^2 \propto x^2 - y^2$.

(b) Find the Compound Interest (CI) on ₹ 1,000 for 2 years at 10% p.a.

(c) The 4^{th} and 7^{th} terms of a G.P. series are respectively 24 and 192. Find the sum of first 10 terms. **(4 × 3 = 12 marks)**

Answer:

(a) As, $x \propto y$

$\therefore \quad \frac{x^2 + y^2}{x^2 - y^2} = \frac{m^2y^2 + y^2}{m^2y^2 - y^2} = \frac{m^2 + 1}{m^2 - 1}$ = a constant

$\therefore \quad x^2 + y^2 \propto x^2 - y^2$

(b) C.I. $= P\left\{\left(1+\frac{i}{100}\right)^n - 1\right\}$

$= 1{,}000\ \{(1 + 0.10)^2 - 1\}$

$= 1{,}000\ (1.21 - 1) = 210$

(c) $t_4 = ar^3 = 24$

$t_7 = ar^6 = 192$

$\Rightarrow \quad \frac{ar^6}{ar^3} = \frac{192}{24}$

$\Rightarrow \quad r^3 = 8$

$\Rightarrow \quad r = 2$

$\therefore \quad a = \frac{24}{2 \times 2 \times 2} \Rightarrow a = 3$

Sum of first 10 term $= \frac{a(r^n - 1)}{r - 1}$

$= \frac{3(2^{10} - 1)}{2 - 1}$

$= 3{,}069$

2018 - Dec [3] Answer the following questions:

(a) The monthly salaries of two persons are in the ratio 7 : 5. If each receives an increase of ₹ 300 in salary, the ratio becomes 25 : 18. Find the respective salaries.

(b) Compute compound interest on ₹ 100 for 2 years at 10% p.a.

(c) The first term of an A.P. is 1 and the sum of its first 10 terms is 100. Find the sum of its first 20 terms. **(4 × 3 = 12 marks)**

Answer:

(a) Let the common multiple be x. So, original salaries will be ₹ 7x and 5x. According to given condition,

$\frac{7x + 300}{5x + 300} = \frac{25}{18}$ $\quad$ 18 (7x + 300) = 25 (5x + 300)

126x + 5,400 = 125x + 7,500 $\quad$ x = 2,100

So, Original Salaries are 7x = 7 × 2,100 = ₹ 14,700 and 5x = 5 × 2,100 = ₹ 10,500

(b) Compound Interest (C.I.) = Amount (A) Principal (P)

= $P [1 + i]^n - P = P [(1 + i)^n - 1]$

= $100 [(1 + 0.1)^2 - 1] = 100 [1.21 - 1]$

= 100 × 0.21 = ₹ 21

In the above calculation, i = rate of interest = r / 100 = 10 / 100 = 0.1

(c) For an Arithmetic Progression, $S_n = n/2 [2a + (n - 1)d]$

100 = (10 / 2) [2(1) + (10 - 1)d]

100 = 5 [2 + 9d]

20 = [2 + 9d]

9d = 18

d = 2..... (1)

Now, S_{20} = (20/2) [2 (1) + (20 - 1) (2)].....[from (1)]

= 10 [2 + 38] = 10 × 40 = 400

2019 - June [3] Answer the questions:

(a) A variable quantity y is equal to sum of two quantities, one of which varies directly as x and the other varies inversely as x. if y = 11 when x = 1 and y = 13 when x = 2, find y when x = 3.

(b) In some years ₹ 1,500 becomes ₹ 1,980 at 8% simple interest. Find the number of years.

(c) If the sum of the 3rd and 4th terms of a G.P. be 60 and that of the 6th and 7th terms be 480, find the 10th term of the G.P. **(4 × 3 = 12 marks)**

Answer:

(a)

$Y = K(q_1 + q_2)$

Now $q_1 = ax$

$q_2 = \frac{b}{x}$

$\therefore \quad Y = K\left(ax + \frac{b}{x}\right)$

$Y = Kax + \frac{Kb}{x}$

if $Ka = A$ and $Kb = B$

then, $Y = Ax + \frac{B}{x}$

Putting $x = 1$, $Y = 11$ we have,

$11 = A \times 1 + \frac{B}{1}$

or $A+B = 11$ ---------- (1)

Putting $x = 2$, $Y = 13$ we have,

$13 = A \times 2 + \frac{B}{2}$

$2A + \frac{B}{2} = 13$

or $4A + B = 26$ ---------- (2)

Subtracting (1) from (2)

$4A + B = 26$

$A + B = 11$

(-) (-) (-)

$3A = 15$

$A = 5$

$B = 11 - 5 = 6$

Now $y = Ax + \frac{B}{x}$

$Y = 5x + \frac{6}{x}$

If $x = 3$, $y = (5 \times 3) + \left(\frac{6}{3}\right)$

$y = 15 + 2 = 17$

(b) $A = P + SI$

$A = P + \frac{Prt}{100}$

$1980 = 1{,}500 + \frac{1{,}500 \times 8 \times t}{100}$

$1980 = 1{,}500 + 120t$

$t = 4$ years

(c) $ar^2 + ar^3 = 60$

$ar^5 + ar^6 = 480$

$ar^2 (1 + r) = 60$ _____ (1)

$ar^5 (1 + r) = 480$ _____ (2)

Dividing (2) by (1)

$\frac{ar^5(1+r)}{ar^2(1+r)} = \frac{480}{60}$

$r^3 = 8$

$r^3 = 2^3$

$r = 2$

$a(2)^2 + a(2)^3 = 60$

$4a + 8a = 60$

$12a = 60$

$a = 5$

10^{th} term $= ar^9 = 5 \times (2)^9 = 2{,}560$

2019 - Dec [3] Answer the questions:

(a) There are 25 members of a student council in a college and the ratio of the number of boys to the number of girls is 3:2. How many more boys should be added to the council so that the ratio of the number of boys to the number of girls is 9:5? **(1 mark)**

(b) What sum of money will amount to ₹3704.40 in 3 years at 5% compound interest? **(1 mark)**

Answer:

(a) Let common multiple be x.

So, original number of boys = 3x and original number of girls = 2x

Now, 3x + 2x = 25

5x = 25

x = 5

Original number of boys = 3x = 3 × 5 = 15 and original number of girls = 2x = 2 × 5 = 10

Let additional boys required be B

According to given condition,

(15 + B)/10 = 9/5

75 + 5B = 90

5B = 15

B = 3

3 more boys should be added to the Council to satisfy required condition.

(b) Amount = $P [\{1+(r/100)\}^3]$

$3704.40 = P [\{1+(5/100)\}^3]$

3704.40 = P [1.157625]

P = ₹3200/-

Required sum of money is ₹3200/-

TOPIC NOT YET ASKED BUT EQUALLY IMPORTANT FOR EXAMINATION

PRACTICAL QUESTIONS

Q.1 While on vacation Priya's diesel purchases were: 10 litres @ ₹ 35.5; 10 litres @ ₹ 36.9; 8 litres @ ₹ 37.5; 14 litres @ ₹ 34. What average price per litre did she pay for the diesel?

Answer :

Purchases in Litres	Rate (₹)	Total Expenditure (₹)
(1)	(2)	(3) = (2) × (1)
10	35.5	355
10	36.9	369
8	37.5	300
14	34	476
42		1,500

Average price per litre of diesel = $\frac{1500}{42}$ = ₹ 35.71

Q.2 A man invested his savings as follows:

₹ 10,000 in Post Office Savings Bank at 8% p.a.
₹ 6,000 in a National Bank at 7% p.a.
₹ 4,000 in a Private Firm at 10% p.a.

Find the average rate of interest per cent p.a.

Answer :

Investment	Amount ₹	Rate of return %	Return ₹
	(1)	(2)	**(3) = (1) x (2)**
Post office	10,000	8	800
National Bank	6,000	7	420
Private Firm	4,000	10	400
	20,000		1,620

Average rate of interest in % = $\frac{1,620 \times 100}{20,000}$ = 8.10%

Q.3 Speed of a car to go up a hill is 10 km per hour and to go down is 20 km per hour. Compute its average speed.

Answer :

Here in the above situation, HM is the appropriate mean.

Average speed = $\frac{2}{\frac{1}{10} + \frac{1}{20}}$

= $\frac{2}{0.1 + 0.05}$

= $\frac{2}{0.15}$ = 13.333 = $13^{1}/_{3}$.

Q.4 In a firm out of 1200 employees 650 are males and rest are females. Average monthly wages of males is ₹ 4,500 and of females is ₹ 5,000. Find the average wage paid by the firm.

Answer :

Sex	No. of employees	Average monthly wages (₹ in)	Total wages paid (₹ in)
	(1)	(2)	(3) = (2) × (1)
Male	650	4,500	29,25,000
Female	550	5,000	27,50,000
	1,200		56,75,000

Average wage paid by the firm = $\frac{56,75,000}{1,200}$

= 4,729.17 (approx)

₹ 4,729.17 approx.

Q.5 X, Y, Z are three children. Y was born when X was 4 years 7 months old and Z was born when Y was 3 years 4 months old. Find their average age when Z was 5 years 2 months old.

Answer :

Person	Age in year	Age in month
X	4 Years 7 months	4 × 12 + 7 = 55
Y	3 years 4 months	3 × 12 + 4 = 40
Z	5 years 2 months	5 × 12 + 2 = 62

We have to find average age when z was 5 years 2 months old

∴ Y's age at this moment = 40 + 62 = 102

Similarly X's age at this moment = 55 + 40 + 62 = 157

∴ Average age in month = $\frac{102 + 157 + 62}{3}$ = 107

and Average age in year = 8 years 11 months

Q.6 The mean of 3 numbers is 15. With inclusion of a fourth number, the mean becomes 17. Find the included number.

Answer :

We know $\bar{x} = \frac{\sum x}{n}$

$15 = \frac{\sum x}{3}$

$\sum x = 15 \times 3 = 45$

Let the fourth number include be x

$$\text{New mean} = \frac{45 + x}{4}$$

$$17 = \text{New mean} = \frac{45 + x}{4}$$

$45 + x = 68$

$X = 68 - 45$

$= 23$

$\therefore$ included number = 23**.**

Q.7 Monthly rainfall from June to September of a certain year was 12.5 cm. 27.04 cm, 20.05 cm and 6.29 cm respectively. Find the average daily rainfall during these four months.

Answer :

Monthly Rainfall in cm no. of days in respective month

Month	Rainfall in c.m.	No. of days in respective month
June	12.5	30
July	27.04	31
August	20.05	31
September	6.29	30
	65.88	122

Average days = $\frac{122}{4} = 30.5$

Average rainfall in month = $\frac{65.88}{4} = 16.47$

$\therefore$ Average daily rainfall during these four month

$= \frac{16.47}{30.5} = 0.54$ cm.

Q.8 A person drove his car for 20 km. at an average speed of 25 km. per hour. At what average speed must he drive for the next 20 km., if his average speed for the whole distance is to be 30 km. per hour?

Answer :

Time taken for 1st 20 Km = $\frac{20}{25} = 0.8$

let average speed for next 20 km be x then, time taken for next 20 km = $\frac{20}{x}$

$$\text{Average time} = \frac{0.8 + \frac{20}{x}}{2}$$

$$\text{Average distance} = \frac{20 + 20}{2} = \frac{40}{2} = 20$$

$$\text{Average speed} = \frac{\text{Average distance}}{\text{Average time}}$$

$$30 = \frac{20}{\frac{0.8 + \frac{20}{x}}{2}}$$

$$30 = \frac{40}{0.8 + \frac{20}{x}}$$

on cross multiplication

$$4 = 2.4 + \frac{60}{x}$$

$$4x = 2.4x + 60$$

$$4x - 2.4x = 60$$

$$1.6x = 60$$

$$x = \frac{60}{1.6} = 37.5 \text{ km/hr.}$$

Q.9 An employer pays wages ₹ 60 per male worker and ₹ 45 per female worker each per day. If he engages 8 male and 4 female workers on some day then find the average wage per worker on that day.

Answer :

Sex	No. of employees	Wages per worker (in ₹)	Total wages paid (in ₹)
	(1)	(2)	(3) = (2) x (1)
Male	8	60	480
Female	4	45	180
	12		660

$$\text{Average wage per worker} = \frac{660}{12} = ₹\ 55$$

Q.10 A person drove his car for first 20 km and then 30 km at an average speed of 20 km and 30 km per hour respectively. At what speed must he drive next 50 km if the average speed of the whole distance of his driving is 40 km per hour?

Answer :

$$\text{Average distance} = \frac{20 + 30 + 50}{3} = 33.333 \text{ km}$$

$$\text{time taken for } 1^{st} \text{ 20 km} = \frac{20}{20} = 1$$

$$\text{time taken for next 30 km} = \frac{30}{30} = 1$$

$$\text{Let speed for next 50 km} = x \text{ km/hr}$$

$$\text{time taken for last 50 km} = \frac{50}{x}$$

$$\text{Average time taken} = \frac{1 + 1 + \frac{50}{x}}{3}$$

$$= \frac{2 + \frac{50}{x}}{3}$$

$$\text{Average speed} = \frac{\text{Average distance}}{\text{Average time}}$$

$$\frac{\frac{33.333}{2+\frac{50}{x}}}{3}$$

$$40 = \frac{99.999}{2+\frac{50}{x}}$$

on cross multiplication

$$80 + \frac{2000}{x} = 99.999$$

$$80x + 2000 = 99.999x$$

$$2000 = 19.999x$$

$$x = 100$$

Average speed is 100 km. per hour.

Q.11 The average score of boys is 60, that of girls is 70 and that of all the candidates is 64 appearing in Mathematics of annual examination. Find the ratio of number of boys and number of girls there. If the total number of candidates appearing in Mathematics is 150, find the number of boys there.

Answer :

Let no. of boys be x, then no. of girls 150 – x

now, $n_1 = x$, $n_2 = 150 - x$

$\overline{x_1} = 60$ $\overline{x_2} = 70$

$\overline{x} = 64$

using combined mean formula

$$\overline{x} = \frac{n_1\overline{x_1} + n_2\overline{x_2}}{n_1 + n_2}$$

$$64 = \frac{x \times 60 + (150 - x) \times 70}{150}$$

$$64 \times 150 = 60x + 10500 - 70x$$

$$9600 = 10500 - 10x$$

$$-900 = -10x$$

$$x = \frac{900}{10}$$

$$= 90$$

$\therefore$ Number of boys = 90

Number of boys (n) = 90.

Q.12 If 16p = 25q, find the duplicate ratio of p to q.

Answer :

16 p = 25 q

$$\frac{p}{q} = \frac{25}{16}$$

Duplicate ratio of p & q

$$\frac{p^2}{q^2} = \frac{25^2}{16^2} = \frac{625}{256}$$

Q.13 The volume of a gas varies as the absolute temperature and inversely as the pressure. When the pressure is 15 units and the temperature is 260 (in absolute units) the volume is 200 cc What will be the volume when the pressure is 18 units and the temperature is 390 (in absolute units)?

Answer:

$$V = \frac{K \times T}{P}$$

Where V = Volume of gas
K = Pressure
T = Temperature
P = Some arbitrary constant

Given P = 15
T = 260
V = 200

$$200 = \frac{K \times 260}{15}$$

$$K = \frac{200 \times 15}{260}$$

Volume, when T = 390 P = 18

$$V = \frac{200 \times 15}{260} \times \frac{390}{18} = 250 \text{ cc}$$

Q.14 If $\frac{5a + 3b}{4a + 5b} = \frac{2}{3}$ find the ratio of a : b

Answer :

$$\frac{5a + 3b}{4a + 5b} = \frac{2}{3}$$

$$3(5a + 3b) = 2(4a + 5b)$$

$$15a + 9b = 8a + 10b$$

$$15a - 8a = 10b - 9b$$

$$7a = b$$

$$\frac{a}{b} = \frac{1}{7}$$

$$\therefore \quad a : b = 1 : 7$$

Q.15 Find the ratio compounded of the duplicate ratio of 2x: 3y and ratio $27y^2$: $8x^3$.

Answer :

Duplicate ratio of $2x : 3y = 2x \times 2x : 3y \times 3y$

$= 4x^2 : 9y^2$

Compound ratio $= 4x^2 \times 27y^2 : 9y^2 \times 8x^3$

$= 108\, x^2\, y^2 : 72y^2x^3$

Note : This can be further reduced to $\frac{108x^2y^2}{72y^2x^3}$

Q.16 Two mixtures contain milk and water in the ratio of 7:2 and 5:1. In what ratio these two mixtures should be mixed so that the resulting mixture may contain milk and water in the ratio 9:2?

Answer:

Milk	7
Water	2

I

Milk	5
Water	1

II

Let x litre is drawn form mixture I and y litre form mixture II.
Then ratio of milk and water in mixture II.

$$\frac{\frac{7x}{9}+\frac{5y}{6}}{\frac{2x}{9}+\frac{y}{6}} = \frac{9}{2}$$

$$\frac{14x}{9}+\frac{10y}{6} = \frac{18x}{9}+\frac{9y}{6}$$

$$\frac{10y}{6}-\frac{9y}{6} = \frac{18x}{9}-\frac{14x}{9}$$

$$\frac{y}{6} = \frac{4x}{9}$$

$$9y = 24x$$

$$\frac{y}{x} = \frac{24}{9}$$

the ratio in which these mixture should be mixed = 24 : 9.

Q.17 The ratio of present age of father to that of his son is 5: 3. Ten years before the ratio was 2 : 1. Find the present ages.

Answer :

Given the ratio of present ages = 5 : 3 and their ratio 10 years before = 2 :1 this data can also be presented as follows:

Person	Present Age	10 Years before age
Father	5x	5x – 10
Son	3x	3x – 10

$\therefore \quad \frac{5x-10}{3x-10} = \frac{2}{1}$

[on cross multiplication]

$5x - 10 = 6x - 20$

$-10 + 20 = 6x - 5x$

$x = 10$

$\therefore$ present age of father is 50 years and, present age of son is 30 years.

Q.18 If x_1, x_2 ....., x_n, be in continued proportion, show that $\frac{x_1}{x_n} = \left(\frac{x_1}{x_2}\right)^{n-1}$.

Answer :

Let $\frac{x_1}{x_2} = \frac{x_2}{x_3} = \text{.....} = \frac{x_{n-1}}{x_n} = K$

Multiplying each ratio

L.H.S. $\frac{x_1}{x_2} \times \frac{x_2}{x_3} \times \text{.....} \frac{x_{n-1}}{x_n} = K^{n-1}$

$\therefore \quad \frac{x_1}{x_2} = K^{n-1}$

R.H.S. $\frac{x_1}{x_2} = K, \quad \therefore \quad \left(\frac{x_1}{x_2}\right)^{n-1} = K^{n-1}$

$$\frac{x_1}{x_n} = \left(\frac{x_1}{x_2}\right)^{n-1}$$

Hence, proved.

Q.19 There are four containers of milk and water in the ratio 2 : 1, 3: 2, 5 : 3 and 7 : 5. A mixture is prepared with equal quantities drawn from the four containers. Find the ratio of milk to water in the final mixture.

Answer :

Let x litre of milk and water is drawn from each container then the ratio of milk of water in final mixture.

$$\Rightarrow \quad \frac{\frac{2}{3}x + \frac{3}{5}x + \frac{5}{8}x + \frac{7}{12}x}{\frac{x}{3} + \frac{2x}{5} + \frac{3x}{8} + \frac{5x}{12}}$$

$$\Rightarrow \quad \frac{x\left(\frac{2}{3} + \frac{3}{5} + \frac{5}{8} + \frac{7}{12}\right)}{x\left(\frac{1}{3} + \frac{2}{5} + \frac{3}{8} + \frac{5}{12}\right)}$$

Taking L.C.M.,

$$\Rightarrow \quad \frac{\frac{80 + 72 + 75 + 70}{120}}{\frac{40 + 48 + 45 + 50}{120}}$$

$$\Rightarrow \quad \frac{297}{120} \times \frac{120}{183}$$

$$\Rightarrow \quad \frac{297}{183} = 99 : 61$$

Q.20 The expenses of a hotel are partly fixed and the rest varies as the number of boarders. When the number of boarders are 450, the expense is ₹ 1,800 and when the number of boarders in 920, the expenses is ₹ 3,210. Find the expenses per head when there are 100 boarders.

Answer :

Let the linear equation be

$y = a + bx$ .....eq. (a)

Where, x = no. of boarders

y = expenses

a = Fixed expense

b = constant multiplier

$1{,}800 = a + 450\,b$ ----------(i)

$3{,}210 = a + 920\,b$ ----------(ii)

Solving equation (i) & eq. (ii) we get,

$a + 450\,b = 1{,}800$

$a + 920\,b = 3{,}210$

$(-) \quad (-) \quad = \quad (-)$

$-470\,b = -1{,}410$

$b = 3$

putting value of b in eq (i)

$a + 450 \times 3 = 1800$

$a = 1{,}800 - 1{,}350$

$a = 450$

Substituting value of a, b in eq (a)

$Y = 450 + 3x$

Where these are 100 boarders, expense =

$y = 450 + 3 \times 100 = 750$

expenses per head $= \frac{750}{100} = ₹ 7.5$

Q.21 Monthly incomes of two persons Ram and Rahim are in the ratio 5:7 and their monthly expenditures are in the ratio 7:11. If each of them saves ₹ 60, per month, determine their monthly incomes.

Answer :

Let monthly income of Ram = 5x

Monthly income of Rahim = 7x

Ram's monthly income	= ₹ 5x
Less : Savings	= 60
Expenditure	5x – 60

again, Rahim's monthly income = ₹ 7x

Less : Savings	= 60
Expenditure	7x – 60

Ratio of their expenditure = 7 : 11

$$\frac{5x - 60}{7x - 60} = \frac{7}{11}$$

$$55x - 660 = 49x - 420$$

$$55x - 49x = -420 + 660$$

$$6x = 240$$

$$x = 40$$

Ram's monthly income = ₹ 5x

= 5 × 40

= ₹ 200

Rahim's monthly income = ₹ 7x

= ₹ 7 × 40

= ₹ 280

Q.22 The ratio of the present age of a father to that of his son is 5 : 3. After Ten years hence the ratio would be 3 : 2. Find their present ages.

Answer :

	Present age	10 years after age
Let the age of Father be	5x	5x + 10
and that of Son be	3x	3x + 10

$$\frac{5x + 10}{3x + 10} = \frac{3}{2}$$

[On cross multiplication]

$10x + 20 = 9x + 30$

$10x - 9x = 30 - 20$

$x = 10$

Present age of father $= 5x = 5 \times 10 = 50$ years

Present age of son $= 3x = 3 \times 10 = 30$ years

Father 50 years, Son 30 years.

Q.23 A dealer mixes tea costing ₹ 8 per kg with tea costing ₹ 7 per kg and thereafter, sells the mixture at ₹ 8 per kg and earns a profit of 7.5% on his sale price. In what proportion does he mix them?

Answer :

Let S.P. be	100
Less :- Profit	7.5
C.P.	92.5

Let proportion of mixture be x : y

$$(8x + 7y) \times \frac{100}{92.5} = 8(x + y)$$

[On cross multiplication]

$(8x + 7y) = 7.4x + 7.4y$

$8x - 7.4x = 7.4y - 7y$

$0.6x = 0.4y$

$$\frac{x}{y} = \frac{0.4}{0.6} = \frac{4}{6} = \frac{2}{3}$$

Q.24 The work done by $n-1$ persons in $n+1$ days is to the work done by n+1 persons in $n+2$ days be in the ratio of 5 : 6; find 'n'.

Answer :

$$\frac{(n-1)(n+1)}{(n+1)(n+2)} = \frac{5}{6}$$

$$\frac{n^2 - 1}{n^2 + 2n + n + 2} = \frac{5}{6}$$

$$\frac{n^2 - 1}{n^2 + 3n + 2} = \frac{5}{6}$$

$$6n^2 - 6 = 5n^2 + 15n + 10$$

$$n^2 = 15n + 16$$

$$n^2 - 15n - 16 = 0$$

$$n^2 - 16n + n - 16 = 0$$

$$n(n - 16) + 1(n - 16) = 0$$

$$n = 16$$

Q.25 Two vessels contain mixtures of milk and water in the proportion 2 : 3 and 4 : 3 respectively. In what proportions should the two mixtures be mixed so as to form a new mixture containing equal quantities of milk and water?

Answer :

Let x litre be drawn form container 1 and y litre be drawn from container 2

$$\frac{\frac{2}{5}x + \frac{4}{7}y}{\frac{3}{5}x + \frac{3}{7}y} = \frac{1}{1}$$

$$\frac{2}{5}x + \frac{4}{7}y = \frac{3}{5}x + \frac{3}{7}y$$

$$\frac{4}{7}y - \frac{3}{7}y = \frac{3}{5}x - \frac{2}{5}x$$

$$\frac{y}{7} = \frac{x}{5}$$

$$5y = 7x$$

$$\frac{x}{y} = \frac{5}{7}$$

The proportion of mixture = 5 : 7.

Q.26 The ratio of the present age of mother to her daughter is 5 : 3. Ten years hence the ratio would be 3 : 2. Find their present ages.

Answer :

Let their present ages be

Person	present age	10 years after age
Mother	5x	5x + 10
Daughter	3x	3x + 10

$$\frac{5x + 10}{3x + 10} = \frac{3}{2}$$

$$10x + 20 = 9x + 30$$

$$x = 10$$

$\therefore$ Present age of mother $= 5x$

$= 5 \times 10$

$= 50$ years

Present age of daughter $= 3x$

$= 3 \times 10$

$= 30$ years

Q.27 Find a mean proportional between 27 and 243.

Answer :

If a, b, c can be in proportion then $\frac{a}{b} = \frac{b}{c}$ then b is said to be in the mean proportion, hence $b^2 = ac$

Mean proportion (b) $= \sqrt{27 \times 243}$

$= \sqrt{6561}$

$= 81$

Q.28 Monthly incomes of two persons are in the ratio 2 : 3 and their monthly expenditures are in the ratio 4 : 7. If each saves ₹ 50 a month, find their monthly incomes and expenditures.

Answer :

Let monthly income be

Person	**Income**	**Saving**	**Expenditure**
1	2x	50	2x – 50
2	3x	50	3x – 50

$$\frac{2x - 50}{3x - 50} = \frac{4}{7}$$

[On cross multiplication]

$$14x - 350 = 12x - 200$$

$$14x - 12x = -200 + 350$$

$$2x = 150$$

$$x = 75$$

Person	Income		Expenditure
1	2x, $2 \times 75 = 150$	$2x - 50$,	$2 \times 75 - 50 = 100$
2	3x, $3 \times 75 = 225$	$3x - 50$,	$3 \times 75 - 50 = 175$

Q.29 If $\frac{4x - 3z}{4c} = \frac{4z - 3y}{3b} = \frac{4y - 3z}{2a}$, show that each ratio is equal to $\frac{x + y + z}{2a + 3b + 4c}$.

Answer :

Adding each ratio $\frac{4x - 3z + 4z - 3y + 4y - 3x}{4c + 3b + 2a}$

$= \frac{x + y + z}{4c + 3b + 2a}$, Proved

Q.30 The average score of girls in HSC examination is 75 and that of boys is 70. The average score of all the candidates in the examination is 72. Find the ratio of number of girls and boys that appeared in the examination.

Answer :

Let total no. of boys be n_1, total no. of girls be n_2 let the total number of students be 100.

i.e. $n_1 + n_2 = 100$

$n_1 = 100 - n_2$

from combined mean formula

$n_1 = 100 - n_2$ $\quad \overline{x_1} = 75$ $\quad \overline{x_2} = 70$

$$\bar{x} = \frac{n_1 \overline{x_1} + n_2 \overline{x_2}}{n_1 + n_2}$$

$$72 = \frac{(100 - n_2) \times 75 + n_2 \times 70}{100}$$

$72 \times 100 = 7{,}500 - 75\, n_2 + 70 n_2$

$7{,}200 = 7{,}500 - 5n_2$

$-300 = -5n_2$

$n_2 = 60$

Then, $n_1 = 100 - 60 = 40$

$\therefore$ the ratio of no. of girls and boys
$= 40 : 60$
$= 2 : 3$

Q.31 If $\frac{a+b}{a-b} = 2$, find the value of $\frac{a^2 - ab + b^2}{a^2 + ab + b^2}$.

Answer :
Given (a+b) : (a−b) = 2:1, we can write a + b = 2 and a−b=1

$$\begin{aligned} a + b &= 2 \\ \underline{a - b} &\ \underline{= 1} \\ 2a &= 3 \end{aligned}$$

$$a = \frac{3}{2}$$

$$a + b = 2$$

$$\frac{3}{2} + b = 2$$

$$b = 2 - \frac{3}{2} = \frac{1}{2}$$

$$\text{Value of } \frac{a^2 - ab + b^2}{a^2 + ab + b^2} = \frac{\frac{9}{4} - \left(\frac{3}{2}\right)\left(\frac{1}{2}\right) + \frac{1}{4}}{\frac{9}{4} + \left(\frac{3}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{4}\right)}$$

$$= \frac{9 - 3 + 1}{9 + 3 + 1}$$

$$= 7 : 13$$

Q.32 What will be the cost price per kg of the mixture of two types of teas, mixed in the ratio 3:2 if the first type is purchased in ₹ 200 per kg and the second in ₹ 300 per kg.?

Answer :
Cost price per kg of the mixture

$$= \frac{3 \times 200 + 2 \times 300}{3 + 2}$$

$$= \frac{600 + 600}{5}$$

$$= \frac{1200}{5}$$

$$= 240$$

Q.33 If $\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{a+b}$, prove that

$$\frac{x(y-z)}{b^2-c^2} = \frac{y(z-x)}{c^2-a^2} = \frac{z(x-y)}{a^2-b^2}.$$

Answer :

Let $\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{a+b} = k$

then $x = (b + c)k$, $y = (c + a)k$, $z = (a + b)k$

$$\frac{x(y-z)}{b^2-c^2} = \frac{k(b+c)[(k(c+a) - k(a-b)]}{b^2-c^2}$$

$$= \frac{(kb+kc)[kc+ka-ka-kb]}{b^2-c^2}$$

$$= \frac{k^2(b+c)(c-b)}{b^2-c^2}$$

$$= \frac{c^2-b^2}{b^2-c^2} = -k^2$$

$$\frac{y(z-x)}{c^2-a^2} = \frac{(c+a)[(k(a+b) - k(b-c)]}{c^2-a^2}$$

$$= \frac{k(c+a)(ka+kb-kb-kc)}{c^2-a^2}$$

$$= \frac{a^2-c^2}{c^2-a^2} = -k^2$$

$$\frac{z(x-y)}{a^2-b^2} = \frac{k(a+b)(kb+kc-kc-ka)}{a^2-b^2}$$

$$= \frac{k^2(b^2-a^2)}{a^2-b^2} = -k^2$$

Hence, $\frac{x(y-z)}{b^2-c^2} = \frac{y(z-x)}{c^2-a^2} = \frac{z(x-y)}{a^2-b^2} = -k^2$ Proved

Q.34 Two vessels contain mixture of milk and water in the proportions 2:3 and 4:3 respectively. In what proportion should the two mixtures be mixed so as to form new mixture containing equal quantities of milk and water?

Answer :

Let x liter be drawn form vessel 1 and y liter form vessel 2

$$\frac{\frac{2}{5}x + \frac{4}{7}y}{\frac{3}{5}x + \frac{3}{7}y} = \frac{1}{1}$$

$$\frac{2}{5}x + \frac{4}{7}y = \frac{3}{5}x + \frac{3}{7}y$$

$$\frac{4}{7}y - \frac{3}{7}y = \frac{3}{5}x - \frac{2}{5}x$$

$$\frac{y}{7} = \frac{x}{5}$$

$$\therefore \frac{x}{y} = \frac{5}{7}$$

the two mixtures should be mixed in the ratio of 5 : 7.

Q.35 If 3, *x* and 27 are in continued proportion, find *x*.

Answer :

$$\frac{3}{x} = \frac{x}{27}$$

$$x^2 = 3 \times 27$$

$$x^2 = 81$$

$$x = 9$$

Q.36 What number is to be added to each term of the ratio 2 : 5 to make it 3 : 4 ?

Answer :

Let x be added to the ratio

$$\frac{2 + x}{5 + x} = \frac{3}{4}$$

$$8 + 4x = 15 + 3x$$

$$4x - 3x = 15 - 8$$

$$x = 7$$

Q.37 If the two numbers 20 and $x + 2$ are in the ratio 2 : 3, find x.

Answer:

$$\frac{20}{x+2} = \frac{2}{3}$$

$$60 = 2x + 4$$

$$60 - 4 = 2x$$

$$56 = 2x$$

$$x = \frac{56}{2} = 28$$

Q.38 If $\frac{\sqrt{a} - \sqrt{b}}{\sqrt{a} + \sqrt{b}} = \frac{1}{2}$, find the value of $\frac{a}{b}$.

Answer:

$$\sqrt{a} - \sqrt{b} = 1 \quad \text{.....(i)}$$

$$\sqrt{a} + \sqrt{b} = 2 \quad \text{.....(ii)}$$

$$2\sqrt{a} = 3$$

$$\sqrt{a} = \frac{3}{2} \qquad a = \frac{9}{4}$$

Putting the value of $\sqrt{a}$ in eq. (i)

$$\frac{3}{2} - \sqrt{b} = 1$$

$$-\sqrt{b} = 1 - \frac{3}{2}$$

$$-\sqrt{b} = \frac{2-3}{2} = -\frac{1}{2}$$

$$\sqrt{b} = \frac{1}{2}, \qquad b = \frac{1}{4}$$

$$\frac{a}{b} = \frac{\frac{9}{4}}{\frac{1}{4}} = \frac{9}{4} \times \frac{4}{1} = 9$$

Q.39 If $\frac{p}{b-c} = \frac{q}{c-a} = \frac{r}{a-b}$, prove that $p + q + r = 0 = pa + qb + rc$.

Answer :

$p = k(b - c)$, $q = k(c - a)$, $r = k(a - b)$

L.H.S. $p + q + r = kb - kc + kc - ka + ka - kb = 0$

R.H.S $pa + qb + rc = (b - c)a + (c - a)b + (a - b)c$

$= ab - ac + bc - ab + ac - bc = 0$

$\therefore \quad p + q + r = 0 = pa + qb + rc$

Q.40 A dealer mixed two varieties of tea having costs ₹ 1,200 and ₹ 2,500 each per kg. in such a way that he can gain 20% by selling the resultant mixture at ₹ 1,800 per kg. Find the proportion in which the two types of tea are mixed.

Answer :

Let the proportion of two types of tea mixed be x : y

then total cost of mixture = 1200 x + 2500 y

resultant mixture is sold at 20% profit

$\therefore (1200x + 2500y)\frac{120}{100} = 1800(x + y)$

$1200x + 2500y = 1500(x + y)$

$2500y - 1500y = 1500x - 1200x$

$1000y = 300x$

$10y = 3x$

$\frac{x}{y} = \frac{10}{3}$

$\therefore$ the ratio of mixture = 10 : 3.

Q.41 The banker's gain on a sum due 10 months hence at 6% per annum is ₹ 50. Find the sum due.

Answer :

$BG = BD - TD$

$BD = A \times n \times i$

$TD = \frac{A \times i \times n}{ni + 1}$

Where A = Sum due

n = no. of years BG = ₹ 50 (given)

r = rate per annum

$$BG = Ani - \frac{A}{ni+1} \times i \times n$$

$$50 = \frac{A \times 10 \times 6}{12 \times 100} - \frac{A \times \frac{6}{100} \times \frac{10}{12}}{\left(\frac{10 \times 6}{12 \times 100} + 1\right)}$$

$$50 = \frac{A}{20} - \frac{A}{\left(\frac{1}{20} + 1\right)} \times \frac{1}{20}$$

$$50 = \frac{A}{20} - \left(1 - \frac{1}{\frac{1}{20} + 1}\right)$$

$$50 = \frac{A}{20} - \left(1 - \frac{20}{20+1}\right)$$

$$50 = \frac{A}{20} - \left(\frac{21-20}{21}\right)$$

$$50 = \frac{A}{420}$$

$A = 420 \times 50 = ₹\ 21,000$

Sum due = ₹ 21,000

Q.42 A man borrowed ₹ 30,000 at 12% simple interest per annum from a bank. After 2 years, he paid ₹ 15,200 to the bank. Find how much he will have to pay after two years more to clear the loan.

Answer :

At first we need to find the amount he is required to pay after 2 years.

$A = P[ni + 1]$ Where $n = 2 i = 12\%$ $P = ₹\ 30,000$

$$= 30,000\left(\frac{2 \times 12}{100} + 1\right)$$

$= ₹\ 37,200$

∴ He is required to pay ₹ 37,200 of this the interest portion is 37,200 – 30,000 = ₹ 7,200. He pays ₹15,200. We assume that he pays ₹ 7,200 towards interest and balance amount towards principal, i.e. ₹ 8,000 (15,200 – 7,200)? the amount he is required to pay after two years to clear the loan.

P = 30,000 – 8,000 = ₹ 22,000

A = 2 years, r = 12 %

$$A = P[ni + 1]$$
$$= 22{,}000\left(\frac{2 \times 12}{100} + 1\right)$$
$$= ₹\ 27{,}280$$

Q.43 Find the present value of ₹ 4,000 due in 6 years if money is worth 5% compounded semi-annually.

Answer:

Given A = ₹ 400 0n = 6 years = 5%

Since interest is compounded semi-annually

$n = 6 \times 2 = 12$ half-yearly

$i = \frac{5}{2} = 2.5\ \%$

$A = P(1 + i)^n$

$$4{,}000 = P\left(1 + \frac{2.5}{100}\right)^{12}$$

$$4{,}000 = P(1.025)^{12}$$

$$\text{or, } P = \frac{4{,}000}{(1.025)^{12}}$$

$$= \frac{4{,}000}{1.34488} = ₹\ 2{,}974$$

Q.44 Equal sum of money were lent to Omkar and Saurabh at 15% simple interest per annum for a period of 3 years and 4 years respectively. If the difference of interest paid by them was ₹ 300, find the sum lent.

Answer :

Simple Interest paid by Omkar = $p \times n \times r$

Where P = principal

r = rate of interest

n = no. of years

$$= P \times 3 \times \frac{15}{100} = 0.45\ p$$

Simple interest paid by Saurabh = $p \times n \times r$

$$= p \times 4 \times \frac{15}{100} = 0.6\ p$$

According to question

$0.6\,p - 0.45\,p = 300$

$0.15\,p = 300$

$P = \frac{300}{0.15} = ₹\ 2{,}000$

Q.45 Compute the Banker's Gain (B.G.) on a bill of ₹ 2,500 due in 6 months at 5% p.a.

Answer :

$BG = BD - TD$

$BD = A \times n \times i, \quad TD = \frac{A}{(ni + 1)} \times i \times n$

Where A = sum due

n = no. of years

i = rate per annum

A = ₹ 2,500 n = 6 months r = 5 %

$BG = A \times n \times i - \frac{A}{(ni + 1)} \times i \times n$

$= 2{,}500 \times \frac{6}{12} \times \frac{5}{100} - 2{,}\frac{500}{\frac{6}{12} \times \frac{5}{100} + 1} \times \frac{5}{100} \times \frac{6}{12}$

$= 62.5 - \frac{62.5}{1.025}$

$= 62.5 - 60.9756 = ₹\ 1.52$ approx

Q.46 A person has deposited ₹ 78,000 in Post Office Monthly Interest Scheme (MIS) after retirement at 8% p.a. Calculate his monthly income.

Answer :

$P = ₹\ 78{,}000$

$r = \frac{8\%}{12} = 0.6667\ \%$

$T = 1$ month

$S.I. = p \times r \times t$

Where p = Principal

r = rate of interest

t = time
S.I. = 78,000 × 0.6667 % × 1
= ₹ 520 (approx)

Q.47 The population of a country increases every year by 2.4% of the population at the beginning of that year. In what time will the population double itself? Answer to the nearest year.

Answer :

Let population at the beginning of year be p
A = 2P r = 2.4%
$A = p(1 + i)^n$
$A = p\left(1 + \frac{2.4}{100}\right)^n$ .....eq (i)
$2p = p(1.024)^n$
$2 = (1.024)^n$ .....(i)
n = 29.22 years

Note : In eq (i) on keeping n = 29, we get value = 1.989 and on keeping n = 30 we get value = 2.037
∴ Difference =
(2.037 – 1.989) = 0.048
0.048 12
0.011 ..... $\frac{12}{0.048} \times 0.011 = 2.75$
on dividing 2.75 / 12 = 0.23

Q.48 Calculate the interest on ₹ 10,000 for 10 years at 10% p.a.

Answer :

Simple interest is to be calculated in the question because it is not mention in the question that interest is compounded annually

Simple interest = p × n × i
Where p = principal amount
n = no. of years
i = rate of interest
A = ₹ 10,000 n = 10 years r = 10 %
$S.I. = 10{,}000 \times 10 \times \frac{10}{100} = 10{,}000$

Q.49 A radio-dealer offers a radio for ₹ 270 cash down or ₹ 30 cash down and 18 equal monthly instalments of ₹ 15 each. Find the rate of simple interest charged.

Answer :

Cash price of radio = ₹ 270
(–) down payment = ₹ 30
Total installment payment = ₹ 240

Each installment of ₹ 15 is to be cleared in 18 months let the rate of simple interest to be charged be i

$$240\ (1 + 18\ i) = 15\ (1 + 17i) + 15\ (1 + 16i) + 15\ (15 + 15i) + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + (1 + 0i)$$

$$240\ (1+ 18i) = 15\ (18 + i\ (17 + 16 + 15 + \ldots\ldots\ldots 0)$$

$$240\ (1 + 18\ i) = 15\ (18 + i \times 153)$$

$$240 + 4{,}320\ i = 270 + 2295\ i$$

$$4{,}320\ i - 2{,}295\ i = 270 - 240$$

$$2{,}025\ i = 30$$

$$i = \frac{30}{2025} = 0.0148$$

∴ yearly interest rate, $i = 0.0148 \times 12$

$= 0.17777$

∴ rate of simple interest charged = 17.77 %

Q.50 The true discount (TD) on a bill for ₹ 2,160 due sometime hence is ₹ 180; find the Banker's gain (BG) on the same bill at the same rate.

Answer :

$$TD = \frac{Ani}{(ni + 1)} \qquad BD = A \times n \times i$$

$$BG = BD - TD$$

Where = TD = True Discount

BD = Bankers Discount

A = Bill Amount

n = rate of interest

BG = Bankers gain

Given TD = 180 A = ₹ 2,160

i.e. $\frac{Ani}{(ni + 1)} = 180$

$$\frac{2160 \times ni}{ni + 1} = 180$$

$$2{,}160 \times ni = 180\ ni + 180$$

$$2{,}160\ ni - 180\ ni = 180$$

$$1980\ ni = 180$$
$$ni = \frac{180}{1980} = 0.0909$$
$$BD = A \times n \times i$$
$$= 2{,}160 \times 0.0909 = 196.36$$
$$\therefore \quad BG = BD - TD$$
$$= 196.36 - 180$$
$$= ₹\ 16.36$$

Q.51 A sum of money becomes double in 20 years at simple interest. Find the number of years by which the sum will be triple.

Answer :

Let sum of money be ₹ P money gets double in 20 years

$$A = (P)\ (ni + i)$$
$$2\ (P) = (P)\ (ni + 1)$$
$$2\ P = P\ (20\ i + 1)$$
$$2 = 20\ i + 1$$
$$2 - 1 = 20\ i$$
$$\frac{1}{20} = i$$
$$i = \frac{1}{20} \times 100 = 5\ \%$$

No. of years in which money gets triple

$$A = (P)\ (ni + 1)$$
$$3(P) = (P)\ \left(n \times \frac{5}{100} + 1\right)$$
$$3 = \frac{5n}{100} + 1$$
$$3 = \frac{5n + 100}{100}$$
$$300 = 5n + 100$$
$$300 - 100 = 5n$$
$$200 = 5n$$
$$n = \frac{200}{5} = 40 \text{ years}$$

Q.52 The simple interest on ₹ 300 at the rate of 4% per annum with that on ₹ 500 at the rate of 3% per annum, both for the same period, is ₹ 162. Find the time period.

Answer:

Let the time period be n

$$S.I. = p \times n \times i$$

p = principal S.I.= Simple interest

i = rate of interest

n = no. of years

$$\text{S.I. at 4\% p.a.} = 300 \times \frac{4}{100} \times n = 12\,n$$

$$\text{S.I. at 3\% p.a.} = 500 \times \frac{3}{100} \times n = 15\,n$$

According to question

$$12\,n + 15\,n = 162$$

$$27\,n = 162$$

$$n = \frac{162}{27} = 6$$

Q.53 At what simple interest rate percent per annum a sum of money will be doubled of itself in 25 years?

Answer :

Let the sum of money be ₹ p

$$A = p\,(ni + 1)$$

$$A = 2\,p\,n = (25\,i + 1)$$

$$2\,p = p\,(25\,i + 1)$$

$$2 = 25\,i + 1$$

$$2 - 1 = 25\,i$$

$$1 = 25\,i$$

$$i = \frac{1}{25}$$

$$i = \frac{1 \times 100}{25} = 4\ \%$$

Q.54 At what rate per annum will a sum of money double itself in 10 years with simple interest?

Answer :

Let the sum of money be ₹ p

$$A = p(ni + 1)$$
$$A = 2p \qquad n = 10$$
$$2p = p(10i + 1)$$
$$2 = 10i + 1$$
$$2 - 1 = 10i$$
$$1 = 10i$$
$$i = \frac{1 \times 100}{10} = 10\ \%$$

Q.55 At the same rate of simple interest a principal amounts to ₹ 2,056 in 4 years and amounts to ₹ 2,248 in 7 years. Find the rate of interest and the principal.

Answer :

$$A = p(ni + 1)$$
$$2056 = p(4 \times i + 1) \quad \text{.....(i)}$$
$$2248 = p(7 \times i + 1) \quad \text{.....(ii)}$$

Dividing eq (ii) by eq (i)

$$\frac{2248}{2056} = \frac{p(7i + 1)}{p(4i + 1)}$$
$$2{,}248(4i + 1) = 2{,}056(7i + 1)$$
$$8{,}992i + 2{,}248 = 14{,}392i + 2{,}056$$
$$2{,}248 - 2{,}056 = 14{,}392i - 8{,}992i$$
$$192 = 5{,}400i$$
$$i = \frac{192}{5{,}400} = 0.035555$$
$$\text{or, } i = 0.035555 \times 100 = 3.55\ \%$$

Putting value of i in eq (i)

$$2{,}056 = p(4 \times 0.0355 + 1)$$
$$2{,}056 = p(0.142 + 1)$$
$$2{,}056 = p(0.142 + 1)$$
$$2{,}056 = p(1.142)$$
$$p = \frac{2{,}056}{1.142} = ₹\ 1{,}800$$

Q.56 What principal will be increased to ₹ 4,600 after 3 years at the rate of 5% per annum simple interest ?

Answer :

$A = p(n\,i + 1)$

$A = ₹\ 4{,}600\ n = 3\ yrs\ \ r = 5\ \%$

$4{,}600 = p(3 \times 0.05 + 1)$

$4{,}600 = p(1.15)$

$p = \frac{4{,}600}{1.15} = ₹\ 4{,}000$

Q.57 An engine without any wagons can run 24 km/hr and its speed is diminished by a quantity varying as the square root of the number of wagons attached to it. With 4 wagons its speed becomes 20 km/hr. Find the maximum number of wagons with which the engine can move.

Answer :

Let n be the no. of wagons

Now speed $= 24 - k\sqrt{n}$

Where k = a constant

Again $20 = 24 - k\sqrt{n}$ (According to Question)

$20 - 24 = -k\sqrt{4}$

$4 = k\sqrt{4}$ or, $4 = 2k$

$K = \frac{4}{2} = 2$

Maximum wagon to be attached so that engine cannot move, i.e. Speed = 0

$0 = 24 - 2\sqrt{n}$

$2\sqrt{n} = 24$

$\sqrt{n} = \frac{24}{2} = 12$, or $n = 144$

$\therefore$ When 144 wagons are attached, engine cannot move.

So, maximum number of wagons with which the engine can move = 144-1 = 143 wagons.

Q.58 If *a* varies as *b* prove that *a + b* varies as *a – b*.

Answer:

Given a varies b

i.e. $a = kb$, where k = a constant

To prove: a + b varies as a – b

i.e. $\frac{a+b}{a-b}$ = a constant [as per variation rule]

L.H.S

$$\frac{a+b}{a-b} = \frac{kb+b}{kb-b} \text{ [putting value of a]}$$

$$= \frac{b(k+1)}{b(k-1)} = \text{a constant}$$

$\therefore$ L.H.S = R.H.S

$\therefore$ a + b varies as a − b

Q.59 If *a + 2b* varies as *a − 2b*, prove that *a* varies as *b*.

Answer:

a + 2b varies as a-2b

$$\text{i.e. } \frac{a+2b}{a-2b} = \text{a constant} = k\text{(let)}$$

To prove :- a varies as b

$$\text{i.e. } \frac{a}{b} = \text{a constant [as per variation rule]}$$

$$\text{L.H.S } \frac{a+2b}{a-2b} = k$$

$$a + 2b = k(a - 2b) \text{ [on cross multiplication]}$$

$$a + 2b = ak - 2bk$$

$$a - ak = -2bk - 2b$$

$$a(1 - k) = -2b(k + 1)$$

$$+ a(k - 1) = +2b(k + 1)$$

$$\frac{a}{b} = \frac{2(k+1)}{(k-1)} = \text{a constant}$$

$\therefore$ L.H.S = R.H.S

a varies as b

Q.60 As the number of units manufactured in a factory is increased from 200 to 300, the total cost of production increases from ₹16,000 to ₹ 20,000. If the total cost of production is partly fixed and other part varies as number of units produced, find the total cost for producing 500 units.

Answer:

Let the total cost function be

$$Y = a + bx$$

Where

Y = total cost

X = no. of units.
a, b are constants.

Total Cost	***production units***
16,000	200
20,000	300

By using above information, we get following equations:

16,000 = a + 200b ---------- (i)

20,000 = a + 300b---------- (ii)

Solving eq(i) and eq(ii) simultaneously

a + 200b = 16,000

a + 300b = 20,000

(-) (-) (-)

+100b = + 4000

$b = \frac{4000}{100} = 40$

Putting value of b in eq (i)

16,000 = a + 200 x 40

a = 16,000 – 8,000

a = 8,000

∴ Total cost function is given by

y = 8,000 + 40x

Hence, Total cost of producing 500 units

y = 8,000 + 40 x 500

= ₹ 28,000

Q.61 If $(a + b)$ varies as $(a - b)$, prove that $a^2 + b^2$ varies as b^2.

Answer:

Given a + b varies as a – b

i.e. $\frac{a + b}{a - b}$ = a constant [As per variation rule]

To prove: $a^2 + b^2$ varies as b^2

$$\text{i.e. } \frac{a^2 + b^2}{b^2} = \text{a constant}$$

$$\text{Proof: } \frac{a + b}{a - b} = k \text{ (let)}$$

$$a + b = ka - kb$$
$$a - ka = -kb - b$$
$$a(1 - k) = -b(k + 1)$$
$$+a(k - 1) = +b(k + 1)$$
$$a(k - 1) = b(k + 1)$$
$$\frac{a}{b} = \frac{k + 1}{k - 1} = \text{a constant}$$

$$\text{Now } \frac{a}{b} = m \text{ (let)}$$

$$\text{or, } a = bm$$

$$\text{L.H.S. : } \frac{a^2 + b^2}{b^2} = \frac{(bm)^2 + b^2}{b^2}$$

$$= m^2 + 1 = \text{ a constant} \qquad \therefore \text{ L.H.S = R.H.S}$$

$\therefore a^2 + b^2$ varies as b^2

Q.62 The expenses of a boarding house are partly fixed and partly varies with the number of boarders. The charge is ₹ 70 per head when there are 20 boarders and ₹ 60 per head when there are 40 boarders. Find the charge per head when there are 50 boarders.

Answer :

Let total change function for boarders = y = a +bx

Where y = cost function

x = no. of boarders

And, a= fixed cost and b = variable cost

According to Question

$20 \times 70 = a + b \times 20$-------- (i)

$40 \times 60 = a + b \times 40$ ------- (ii)

Solving eq(i) and (ii) simultaneously

$a + 20b = 1400$

$a + 40b = 2400$

(-) (-) (-)

$-20b = -1,000$

$b = 50$

Putting value of b in eq (i)

$70 \times 20 = a + 20 \times 50$

$1400 = a + 1000$

$a = 1400 - 1000$

$= 400$

$\therefore$ Total cost function $= 400 + 50\ x$

Total cost for 50 boarders

$= 400 + 50 \times 50$

[putting value of x = 50 in total cost function 50 ascertain]

$= 400 + 2500$

$= ₹\ 2{,}900$

Charge per head for 50 boarders

$= \frac{2900}{50} = ₹\ 58$ per head

Q.63 If *x* varies as *y* then show that $x^2 + y^2$ varies as $x^2 - y^2$.

Answer :

Given: x varies as y

i.e. $\frac{x}{y}$ = a constant [As per variation rule]

To prove: $x^2 + y^2$ varies as $x^2 - y^2$

i.e. $\frac{x^2 + y^2}{x^2 - y^2}$ = a constant

L.H.S: $\frac{x}{y} = k$ (let)

$x = ky$

$\frac{x^2 + y^2}{x^2 - y^2} = \frac{(ky)^2 + y^2}{(ky)^2 - y^2} = \frac{y^2(k^2 + 1)}{y^2(k^2 - 1)}$ = a constant

$\therefore$ L.H.S= R.H.S

$x^2 + y^2$ varies as $x^2 - y^2$